ISBN 978-9967-27-454-9

Existence, Singleness and Smoothness in the Problem of Navier-Stokes for the Incompressible Fluid with Viscosity

Taalaibek D.Omurov

Doctor of Physics and Mathematics, professor of Z. Balasagyn
Kyrgyz National University,
Bishkek, Kyrgyzstan, E-mail: omurovtd@mail.ru

Abstract. Existence, singleness and smoothness (or conditional-smoothness) in solution of the Navier-Stokes equation is one of the most important problems in mathematics of the millennium [1], which describes the motion of viscous Newtonian fluid and which is a basic in hydrodynamics [6, 12]. Therefore in this work a nonstationary problem for Navier-Stokes of incompressible fluid with viscosity is solved [1].
Keywords: 6^{th} millennium problem, equations Navier-Stokes and Euler, Beale-Kato-Majda, Critical Reynolds number

Preface

The research is devoted to the development of a method for solving 3D Navier-Stokes equations that describe the flow of a viscous incompressible fluid. The study includes a requirements "Navier-Stokes Millennium Problem", as developed method of solution contains a proof of the existence and smoothness of solutions of the Navier-Stokes equations, where laminar flow is separated from the turbulent flow when the critical Reynolds number: **Re** = 2300. The decision is obtained for the velocity and pressure in an analytical form, as required by the "Navier-Stokes problem Millennium". The method of solution is supported by examples for different viscosity ranges corresponding applications.

In sections 4.3, 4.4, 7.2 and paragraphs 5, 6 new law of the pressure distribution has been found. This law is derived from the equation of Poisson type and differs from the known laws of Bernoulli, Darcy at all. Most importantly, the author has opened a special space for the study of the existence and smoothness (including conditional smoothness) equations Navier-Stokes for viscous incompressible fluid. In the case of smoothness a space with the norms of Chebyshev type has been obtained. The weighted space of Sobolev type arises in the case of conditional-smoothness. For brevity, these spaces can be called: Omurov's spaces with different metrics.

K. Jumaliev, Academician, Director of the Institute of Physics NAS Kyrgyz Republic
August 1, 2014

1. Introduction

If to designate components of vectors of speed and external force, as

$$\mathbf{v}(x,t)=[\upsilon_1(x,t),\upsilon_2(x,t),\upsilon_3(x,t)],\ \mathbf{f}(x,t)=[f_1(x,t),f_2(x,t),f_3(x,t)],$$

that corresponding problem Navier-Stokes is represented in a kind

$$\upsilon_{it}+\sum_{j=1}^{3}\upsilon_j\upsilon_{ix_j}=f_i-\frac{1}{\rho}P_{x_i}+\mu\Delta\upsilon_i,(i=\overline{1,3}), \tag{1.1}$$

$$\operatorname{div} v=0,\forall(x,t)\in T=R^3\times[0,T_0], \tag{1.2}$$

$$\upsilon_i\big|_{t=0}=\upsilon_{i0}(x_1,x_2,x_3),\forall(x_1,x_2,x_3)\in R^3, \tag{1.3}$$

$\mu>0$ is kinematic viscosity, ρ is density, Δ is Laplace operator. Here, the condition incompressibility (1.2) fluid it's a the additional equation. Unknown are speed v and pressure P.

The decision of many problems of theoretical and mathematical physics leads to use of various a special weight spaces. In works [7, 8] for the first time a method have been offered, which gives solution of problem Navier-Stokes in $G_\lambda^2(D_0)$. Alternatively, we can consider, e.g., a class of suitable solutions constructed in [8]: $W_\lambda^2(D_0)$ on the basis of lemma K. Friedrichs [15].

To answer this question, in this article the following way proposed to solve for the Navier-Stokes equations. For this purpose (1.1) we will transform to a kind

$$\upsilon_{it}+\theta_i=f_i-\frac{1}{\rho}P_{x_i}-\frac{1}{2}Q_{x_i}+\mu\Delta\upsilon_i,(i=\overline{1,3}), \tag{1.4}$$

$$\theta_i=\sum_{j=1}^{3}(\upsilon_j\upsilon_{ix_j}-\frac{1}{2}Q_{x_i}), \tag{1.5}$$

where

$$\begin{cases}\theta_i\big|_{t=0}=\theta_i^0(x_1,x_2,x_3),\forall(x_1,x_2,x_3)\in R^3,\\ Q(x_1,x_2,x_3,t)\equiv\sum_{i=1}^{3}\upsilon_i^2(x_1,x_2,x_3,t);\ \ Q_{x_i}=2\sum_{j=1}^{3}\upsilon_j\upsilon_{jx_i};\ \ Q_{x_i}^0=2\sum_{j=1}^{3}\upsilon_{j0}\upsilon_{j0x_i},(i=\overline{1,3}),\end{cases}$$

without breaking equivalence of system (1.1) and (1.4), (1.5). The received systems (1.4), (1.5) contain unknown functions υ_i, θ_i and pressure P. Here θ_i^0 – known functions because are known $\upsilon_{j0},\upsilon_{j0x_i}$.

The developed method of the decision of systems (1.4) and (1.5) connected with functions $\theta_i,(i=\overline{1,3})$, i.e.

A1) $\operatorname{rot}\tilde{\theta}=0,\tilde{\theta}=(\theta_1,\theta_2,\theta_3)$; $\operatorname{rot} v\neq 0$, or

A2) $\operatorname{div}\tilde{\theta}=0$, $\operatorname{rot} v\neq 0$, or

A3) $\theta_i, (i = \overline{1,3})$ is any functions if, accordingly, as necessary conditions, take place:

a01) $\mathrm{rot}\,\tilde{\theta}^0 = 0, \tilde{\theta}^0 = (\theta_1^0, \theta_2^0, \theta_3^0)$, a02) $\mathrm{div}\,\tilde{\theta}^0 = 0$, a03) $\tilde{\theta}^0$ is any functions.

The work purpose. The main object of this work – the proof existence, singleness and smoothness (or conditional-smoothness) of the problem decision Navier-Stokes for an incompressible fluid with viscosity in cases (A1)-(A3). In the case of smoothness have the space $\tilde{C}^{3,1}_{n=3}(T)$:

$$\begin{cases} v = (\upsilon_1, \upsilon_2, \upsilon_3): \quad \|v\|_{\tilde{C}^{3,1}_{n=3}} = \sum_{i=1}^{3} \|\upsilon_i\|_{\tilde{C}^{3,1}} = \{\sum_{i=1}^{3}\{\sum_{0\le|k|\le 3} \|D^k \upsilon_i\|_C + \|\upsilon_{it}\|_C\}, \\ \tilde{C}^{3,1}_{n=3}(T) \equiv \tilde{C}^{3,3,3,1}_{n=3}(T) \ne C^{3,3,3,1}_{n=3}(T); \quad \upsilon_{i0} \in C^3(R^3),\ (i = \overline{1,3}), \\ k = 0 : D^0 \upsilon_i \equiv \upsilon_i;\ k \ne 0 : D^k \upsilon_i = \dfrac{\partial^{|k|} \upsilon_i}{\partial x_1^{\alpha_1} \partial x_2^{\alpha_2} \partial x_3^{\alpha_3}}; \quad |k| = \sum_{i=1}^{3} \alpha_i,\ (\alpha_i = \overline{0,3}), \end{cases} \tag{1.6$_1$}$$

but in the case of a conditional smoothness – the space $G^1_{n=3}(D_0)$:

$$\begin{cases} \|v\|_{G^1_{n=3}(D_0)} = \sum_{i=1}^{3} \|\upsilon_i\|_{G^1(D_0)} = \{\sum_{i=1}^{3}\{\sum_{0\le|k|\le 3} \|D^k \upsilon_i\|_{C(T)} + \|\upsilon_{it}\|_{L^1}\},\ D_0 = R^3 \times (0, T_0), \\ \upsilon_{i0} \in C^3(R^3),\ (i = \overline{1,3}); \quad \|\upsilon_{it}\|_{L^1} = \sup_{R^3} \int_0^{T_0} |\upsilon_{it}(x_1, x_2, x_3, t)| dt. \end{cases} \tag{1.6$_2$}$$

So as $\upsilon_{i0} \in C^3(R^3)$, then limitation of solution of problem Navier-Stokes (1.1) - (1.3) it is possible to prove and in $W^2_\lambda(D_0)$ – weight space of Sobolev's type:

$$\begin{cases} \|v\|_{W^2_\lambda} = \sum_{i=1}^{3} \|\upsilon_i\|_{\tilde{W}^2_{(\upsilon_i,\lambda)}}, \quad (v = (\upsilon_1, \upsilon_2, \upsilon_3)), \\ \tilde{W}^2_{(\upsilon_i,\lambda)} = \{(x_1, x_2, x_3, t) \in D_0 : D^k \upsilon_i \in L^2; \quad \upsilon_{it} \in L^2_\lambda\}, (i = \overline{1,3}), \\ \|\upsilon_i\|_{\tilde{W}^2_{(\upsilon_i,\lambda)}} = \{\sum_{0\le|k|\le 3} \sup_{R^3} \int_0^{T_0} [D^k \upsilon_i(x_1, x_2, x_3, t)]^2 dt + \sup_{R^3} \int_0^{T_0} \lambda(t) |\upsilon_{it}(x_1, x_2, x_3, t)|^2 dt\}^{\frac{1}{2}}. \end{cases} \tag{1.6$_3$}$$

It is known that from uniform convergence of sequence continuous functions on [a, b] is followed by its convergence on the average on [a, b]. Therefore, so as norm: $\|v\|_{W^2_\lambda}$ it is subordinated to norm $\|v\|_{\tilde{C}^{3,1}_{n=3}}$, that is natural describe an analytical solution in $\tilde{C}^{3,1}_{n=3}(T)$. Hence, gives also feasibility to construct the decision in $W^2_\lambda(D_0)$, the converse is not true.

The scientific value. Actually at use of offered transformations linearization of equations

Navier-Stokes occurs in the integrated form without the requirement of additional conditions. Consequently, the solution of the resulting integral equations possesses the same properties as the solution of initial value problems for Navier-Stokes. The analytical decision, obvious, is regular in concerning viscosity factor $\mu > 0$ and in many respects simplifies carrying out of the analysis in mathematical and physical sense [6, 11 and 12].

In a case $0 < \mu < 1$ the current is considered with very small viscosity, i.e. in viscous liquids, when force of friction is very small, than forces of inertia [11, 12]. Here Reynolds number is very great (**Re** ≥ 2300) there is a border layer in which viscosity influence is concentrated. Therefore the analytical methods of the decisions of a problem Navier-Stokes allow to reach full understanding of physics of turbulence [4, 11 and 12].

In a case $1 \leq \mu = \mu_0 = \mathrm{const} < \infty$ the current is considered with average size of viscosity [12]. Therefore in a case when convective acceleration is not equal to zero then there are problems connected with methods of integration of the equations of Navier-Stokes in their general view.

Our problem does not include a derivation of an equation in a physical meaning, since there is a big amount of works reflecting these questions [3, 4, 6, 11 and 12].

2. Fluid with very small Viscosity by Condition (A_1)

In this paragraph and in the subsequent points with the specified restrictions at the entrance data, the strict substantiation of compatibility of systems (1.4), (1.5) will be given with very small viscosity $0 < \mu < 1$. In the limiting case of very small frictional forces (for large Reynolds numbers), the solution of the Navier-Stokes equations has such properties that the flow field can be divided into two regions [12]. The friction manifests itself in a thin layer. The flow in the outer region does not depend on friction forces, it is free from rotation of the particles, and hence, it is described by Euler equations. Therefore, in this section we study the behavior of the solution of the Navier-Stokes equations when the viscosity tends to zero.

2.1. Fluid with the condition (A_1)

Let functions $\theta_i^0, (i = \overline{1,3})$ satisfy to a condition (a_{01}). Then relatively $\theta_i, (i = \overline{1,3})$ we suppose a condition (A_1) and

$$\mathrm{div}\, f \neq 0 \tag{2.1}$$

where from system (1.4) and (1.5), accordingly we will receive following systems

$$\upsilon_{it} + \theta_{x_i} + \frac{1}{2} Q_{x_i} = f_i - \frac{1}{\rho} P_{x_i} + \mu \Delta \upsilon_i, (i = \overline{1,3}), \tag{2.2}$$

$$\theta_i = \theta_{x_i} : \ \theta_{x_i} = \sum_{j=1}^{3} (\upsilon_j \upsilon_{ix_j} - \frac{1}{2} Q_{x_i}), (i = \overline{1,3}). \tag{2.3}$$

Theorem 1. Let conditions (1.2), (1.3), (A_1) and (2.1) are satisfied. Then systems (2.2) and (2.3) equivalent will be transformed to a kind

$$\begin{cases} \Delta J = -F_0, \quad [F_0(x_1,x_2,x_3,t) \equiv -\sum_{i=1}^{3} f_{ix_i}; \quad J(x_1,x_2,x_3,t) \equiv \frac{1}{\rho}P + \frac{1}{2}Q + \theta], \\ \upsilon_{it} = f_i + \mu\Delta\upsilon_i - J_{x_i}, (i = \overline{1,3}), \\ \Delta\theta = -\psi^0, \quad [\psi^0(x_1,x_2,x_3,t) \equiv -\sum_{i=1}^{3} \psi_{ix_i}], \\ \frac{1}{\rho}P + \frac{1}{2}Q = -\theta + \frac{1}{4\pi}\int_{R^3} F_0(s_1,s_2,s_3,t)\frac{ds_1ds_2ds_3}{r}, (r = \sqrt{\sum_{i=1}^{3}(x_i - s_i)^2}). \end{cases} \quad (2.4)$$

Thereby the problem (1.1) - (1.3) has the only solution which satisfies to a condition (1.2).

Proof. Proof of the theorem 1 consists of four stages.

1) From system (2.2) it is visible, if the 1-equation (2.2, *i=1*) it is differentiated on x_1, 2-equation on x_2 (2.2, *i=2*), 3-equation on x_3 (2.2, *i=3*), and based on the formula:

$$\begin{cases} \sum_{i=1}^{3}\frac{\partial}{\partial x_i}(2.2); \quad \operatorname{div} v = 0: \\ \frac{\partial}{\partial t}(\sum_{i=1}^{3}\upsilon_{ix_i}) = 0; \quad \mu\sum_{i=1}^{3}\frac{\partial^2}{\partial x_i^2}(\upsilon_{1x_1} + \upsilon_{2x_2} + \upsilon_{3x_3}) = 0, \\ -\sum_{i=1}^{3} f_{ix_i} \equiv F_0, \\ \sum_{i=1}^{3}\frac{\partial}{\partial x_i}[\frac{1}{\rho}P_{x_i} + \theta_{x_i} + \frac{1}{2}Q_{x_i}] \equiv \Delta[\frac{1}{\rho}P + \theta + \frac{1}{2}Q], \end{cases} \quad (2.5)$$

from here we will receive the equation of Poisson [13]:

$$\Delta[\frac{1}{\rho}P + \theta + \frac{1}{2}Q] = -F_0,$$

i.e.

$$\begin{cases} \Delta J = -F_0, \\ J = \frac{1}{4\pi}\int_{R^3} F_0(s_1,s_2,s_3;t)\frac{ds_1ds_2ds_3}{r}, \\ J_{x_i} = \frac{1}{4\pi}\int_{R^3} F_0(x_1+\tau_1, x_2+\tau_2, x_3+\tau_3;t)\frac{\tau_i}{\sqrt{(\tau_1^2+\tau_2^2+\tau_3^2)^3}}d\tau_1 d\tau_2 d\tau_3, \\ s_i - x_i = \tau_i, (i = \overline{1,3}). \end{cases} \quad (2.6)$$

so as

$$\frac{1}{\rho}P + \theta + \frac{1}{2}Q \equiv J. \quad (2.7)$$

The algorithm in which we received the Poisson equation (2.6), for the sake of brevity we call "algorithm poissonization system", hereinafter APS. Therefore, if J – the decision of the equation (2.6), then substituting

$$\frac{1}{\rho}P_{x_i}+\frac{1}{2}Q_{x_i}+\theta_{x_i}\equiv J_{x_i},(i=\overline{1,3}),\tag{2.8}$$

in (2.2), we have

$$\begin{cases}\upsilon_{it}=\Phi_i+\mu\Delta\upsilon_i,(i=\overline{1,3}),\\ \Phi_i(x_1,x_2,x_3,t)\equiv f_i-J_{x_i},(i=\overline{1,3}),\ \sum_{i=1}^{3}\Phi_{ix_i}\equiv F_0+\Delta J=0,\ \forall(x_1,x_2,x_3,t)\in T,\end{cases}\tag{2.9}$$

i.e. system (2.2) it is equivalent by (2.9). This means that the system (2.2) is converted in linear an inhomogeneous equation of heat conduction. Here the equations (2.6), (2.9) is there are first and second equations of system (2.4).

2) From the received results follows that the system (1.1) is transformed in the linear equations of heat conductivity with a condition of Cauchy. Consequently, Cauchy problem with sufficiently smooth initial data $t=0$ in the class of bounded functions is solvable [13, 14]. Accordingly, the problem of the Navier-Stokes equations has a single, conditional smooth solution [8] in the space $G^{1}_{n=3}(D_0)$.

Really from system (2.9), follows

$$\upsilon_i=\frac{1}{8(\sqrt{\pi\mu t})^3}\int_{R^3}\exp(-\frac{r^2}{4\mu t})\upsilon_{i0}(s_1,s_2,s_3)ds_1ds_2ds_3+\frac{1}{8\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-\frac{r^2}{4\mu(t-s)})\frac{1}{\sqrt{(\mu(t-s))^3}}\times$$
$$\times\Phi_i(s_1,s_2,s_3,s)ds_1ds_2ds_3ds=\frac{1}{\sqrt{\pi^3}}\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\upsilon_{i0}(x_1+2\tau_1\sqrt{\mu t},x_2+2\tau_2\sqrt{\mu t},x_3+2\tau_3\times$$
$$\times\sqrt{\mu t})d\tau_1d\tau_2d\tau_3+\frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Phi_i(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+\tag{2.10}$$
$$+2\tau_3\sqrt{\mu(t-s)};s)d\tau_1d\tau_2d\tau_3ds\equiv H_i(x_1,x_2,x_3,t),$$
$$s_i-x_i=2\tau_i\sqrt{\mu t};s_i-x_i=2\tau_i\sqrt{\mu(t-s)},(i=\overline{1,3}).$$

All H_i – is known functions. The found decision (2.10) satisfies system (2.9).

Really, considering partial derivative systems (2.10):

$$\begin{cases}(0,1)\ni\mu;\ \forall(x_1,x_1,x_1,t)\in T:\\ \upsilon_{ix_j}=\frac{1}{\sqrt{\pi^3}}\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\upsilon_{i0h_j}(x_1+2\tau_1\sqrt{\mu t},x_2+2\tau_2\sqrt{\mu t},x_3+2\tau_3\sqrt{\mu t})d\tau_1d\tau_2d\tau_3+\\ +\frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Phi_{il_j}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\times\\ \times\sqrt{\mu(t-s)};s)d\tau_1d\tau_2d\tau_3ds,\end{cases}$$

$$
\begin{cases}
\upsilon_{ix_j^2} = \dfrac{1}{\sqrt{\pi^3}} \displaystyle\int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\upsilon_{i0h_j^2}(x_1+2\tau_1\sqrt{\mu t}, x_2+2\tau_2\sqrt{\mu t}, x_3+2\tau_3\sqrt{\mu t})d\tau_1 d\tau_2 d\tau_3 + \\
+\dfrac{1}{\sqrt{\pi^3}} \displaystyle\int_0^t \int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Phi_{il_j^2}(x_1+2\tau_1\sqrt{\mu(t-s)}, x_2+2\tau_2\sqrt{\mu(t-s)}, x_3+2\tau_3 \times \\
\times\sqrt{\mu(t-s)};s)d\tau_1 d\tau_2 d\tau_3 ds, \\
\forall (x_1,x_2,x_3)\in R^3;\ \ t\in(0,T_0]: \\
\upsilon_{it} = \dfrac{1}{\sqrt{\pi^3}} \displaystyle\int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\times(\sum_{j=1}^3 \sqrt{\mu}\,\frac{\tau_j}{\sqrt{t}}\upsilon_{i0h_j}(x_1+2\tau_1\sqrt{\mu t}, x_2+2\tau_2\sqrt{\mu t}, x_3+ \\
+2\tau_3\sqrt{\mu t})d\tau_1 d\tau_2 d\tau_3 + \Phi_i + \dfrac{1}{\sqrt{\pi^3}} \displaystyle\int_0^t \int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\sum_{j=1}^3 \sqrt{\mu}\,\frac{\tau_j}{\sqrt{t-s}}\Phi_{il_j}(x_1+2\tau_1\sqrt{\mu(t-s)}, \\
x_2+2\tau_2\sqrt{\mu(t-s)}, x_3+2\tau_3\sqrt{\mu(t-s)};s)d\tau_1 d\tau_2 d\tau_3 ds, \\
h_j = x_j+2\tau_j\sqrt{\mu t};\ l_j = x_j+2\tau_j\sqrt{\mu(t-s)},\ \ (i=\overline{1,3};\ j=\overline{1,3}),
\end{cases}
\tag{2.11}
$$

and substituting (2.11) in (2.10), we have

$$
\begin{cases}
\upsilon_i\big|_{t=0} = \upsilon_{i0}(x_1,x_2,x_3),\ \ \forall(x_1,x_2,x_3)\in R^3;\ \ (0,1)\ni\mu;\ \ \forall(x_1,x_2,x_3)\in R^3;\ \ t\in(0,T_0]: \\
0 = \upsilon_{it} - \Phi_i - \mu\Delta\upsilon_i \equiv \dfrac{1}{\sqrt{\pi^3}} \displaystyle\int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\times(\sum_{j=1}^3 \sqrt{\mu}\,\frac{\tau_j}{\sqrt{t}}\upsilon_{i0h_j}(x_1+2\tau_1\sqrt{\mu t}, x_2+2\tau_2\sqrt{\mu t}, x_3+ \\
+2\tau_3\sqrt{\mu t}))d\tau_1 d\tau_2 d\tau_3 + \Phi_i + \dfrac{1}{\sqrt{\pi^3}} \displaystyle\int_0^t \int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\times(\sum_{j=1}^3 \sqrt{\mu}\,\frac{\tau_j}{\sqrt{t-s}}\Phi_{il_j}(x_1+2\tau_1\sqrt{\mu(t-s)}, \\
x_2+2\tau_2\sqrt{\mu(t-s)}, x_3+2\tau_3\sqrt{\mu(t-s)};s))d\tau_1 d\tau_2 d\tau_3 ds - \Phi_i - \mu\{\dfrac{1}{\sqrt{\pi^3}} \displaystyle\int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\times \\
\times\Delta\upsilon_{i0}(x_1+2\tau_1\sqrt{\mu t}, x_2+2\tau_2\sqrt{\mu t}, x_3+2\tau_3\sqrt{\mu t})d\tau_1 d\tau_2 d\tau_3 + \dfrac{1}{\sqrt{\pi^3}} \displaystyle\int_0^t \int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Delta\Phi_i(x_1+ \\
+2\tau_1\sqrt{\mu(t-s)}, x_2+2\tau_2\sqrt{\mu(t-s)}, x_3+2\tau_3\sqrt{\mu(t-s)};s)d\tau_1 d\tau_2 d\tau_3 ds\} = \\
= \dfrac{1}{\sqrt{\pi^3}} \displaystyle\int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2))(\sum_{j=1}^3 \sqrt{\mu}\,\frac{\tau_j}{\sqrt{t}}\upsilon_{i0h_j}(x_1+2\tau_1\sqrt{\mu t}, x_2+2\tau_2\sqrt{\mu t}, x_3+2\tau_3\sqrt{\mu t}))\times \\
\times d\tau_1 d\tau_2 d\tau_3 + \dfrac{1}{\sqrt{\pi^3}} \displaystyle\int_0^t \int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2))(\sum_{j=1}^3 \sqrt{\mu}\,\frac{\tau_j}{\sqrt{t-s}}\Phi_{il_j}(x_1+2\tau_1\sqrt{\mu(t-s)}, x_2+2\tau_2\times \\
\times\sqrt{\mu(t-s)}, x_3+2\tau_3\sqrt{\mu(t-s)};s))d\tau_1 d\tau_2 d\tau_3 ds - \dfrac{1}{2}\sqrt{\mu}\{\dfrac{1}{\sqrt{\pi^3}} \displaystyle\int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\frac{1}{\sqrt{t}}\upsilon_{i0h_1^2}(x_1+ \\
+2\tau_1\sqrt{\mu t}, x_2+2\tau_2\sqrt{\mu t}, x_3+2\tau_3\sqrt{\mu t})d(x_1+\sqrt{\mu t})d\tau_2 d\tau_3 + \dfrac{1}{\sqrt{\pi^3}} \displaystyle\int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\frac{1}{\sqrt{t}}\times \\
\times\upsilon_{i0h_2^2}(x_1+2\tau_1\sqrt{\mu t}, x_2+2\tau_2\sqrt{\mu t}, x_3+2\tau_3\sqrt{\mu t})d\tau_1 d(x_2+2\tau_2\sqrt{\mu t})d\tau_2 d\tau_3 + \\
+\dfrac{1}{\sqrt{\pi^3}} \displaystyle\int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\frac{1}{\sqrt{t}}\upsilon_{i0h_3^2}(x_1+2\tau_1\sqrt{\mu t}, x_2+2\tau_2\sqrt{\mu t}, x_3+2\tau_3\sqrt{\mu t})d\tau_1 d\tau_2 d(x_3+
\end{cases}
$$

$$
\begin{cases}
+2\tau_3\sqrt{\mu t}\,)+\dfrac{1}{\sqrt{\pi^3}}\int\limits_0^t\dfrac{1}{\sqrt{t-s}}[\int\limits_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Phi_{il_1^2}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+\\
+2\tau_3\sqrt{\mu(t-s)};s)d(x_1+2\tau_1\sqrt{\mu(t-s)})d\tau_2 d\tau_3+\int\limits_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Phi_{il_2^2}(x_1+2\tau_1\sqrt{\mu(t-s)},\\
x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)d\tau_1 d(x_2+2\tau_2\sqrt{\mu(t-s)})d\tau_3+\int\limits_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\times\\
\times\Phi_{il_3^2}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)\times\\
\times d\tau_1 d\tau_2 d(x_3+2\tau_3\sqrt{\mu(t-s)})]ds\}=\dfrac{1}{\sqrt{\pi^3}}\int\limits_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))(\sum\limits_{j=1}^{3}\sqrt{\mu}\,\dfrac{\tau_j}{\sqrt{t}}\upsilon_{i0h_j}(x_1+2\tau_1\sqrt{\mu t},\\
x_2+2\tau_2\sqrt{\mu t},x_3+2\tau_3\sqrt{\mu t}\,))d\tau_1 d\tau_2 d\tau_3+\dfrac{1}{\sqrt{\pi^3}}\int\limits_0^t\int\limits_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\times(\sum\limits_{j=1}^{3}\sqrt{\mu}\,\dfrac{\tau_j}{\sqrt{t-s}}\Phi_{il_j}(x_1+\\
+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s))d\tau_1 d\tau_2 d\tau_3 ds-\\
-\dfrac{1}{\sqrt{\pi^3}}\int\limits_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))(\sum\limits_{j=1}^{3}\sqrt{\mu}\,\dfrac{\tau_j}{\sqrt{t}}\upsilon_{i0h_j}(x_1+2\tau_1\sqrt{\mu t},x_2+2\tau_2\sqrt{\mu t},x_3+2\tau_3\sqrt{\mu t}\,))\times\\
\times d\tau_1 d\tau_2 d\tau_3-\dfrac{1}{\sqrt{\pi^3}}\int\limits_0^t\int\limits_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\times(\sum\limits_{j=1}^{3}\sqrt{\mu}\,\dfrac{\tau_j}{\sqrt{t-s}}\Phi_{il_j}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\times\\
\times\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s))d\tau_1 d\tau_2 d\tau_3 ds\}=0,
\end{cases}
\qquad (*)
$$

on the right side integrals of the formula (*) the integration method in parts is used. That it was required to prove.

Further we will show that (2.10) satisfies (1.2). For this purpose considering partial derivatives of 1st order and summarizing, with taking into account (1.2), we have

$$
\begin{cases}
0=\sum\limits_{i=1}^{3}\upsilon_{x_i}(x_1,x_2,x_3,t)=\dfrac{1}{\sqrt{\pi^3}}\int\limits_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\sum\limits_{i=1}^{3}\dfrac{\partial}{\partial x_i}\upsilon_{i0}(x_1+2\tau_1\sqrt{\mu t},x_2+2\tau_2\sqrt{\mu t},x_3+\\
+2\tau_3\sqrt{\mu t}\,)d\tau_1 d\tau_2 d\tau_3+\dfrac{1}{\sqrt{\pi^3}}\int\limits_0^t\int\limits_{R^3}\exp[-(\tau_1^2+\tau_2^2+\tau_3^2)]\sum\limits_{i=1}^{3}\dfrac{\partial}{\partial x_i}\Phi_i(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+\\
+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)d\tau_1 d\tau_2 d\tau_3 ds=0,\\
\sum\limits_{i=1}^{3}\dfrac{\partial}{\partial x_i}\upsilon_{i0}=0;\quad \sum\limits_{i=1}^{3}\dfrac{\partial}{\partial x_i}\Phi_i\equiv F_0+\Delta J=0.
\end{cases}
$$

The system (2.10) satisfies to the equation (1.2).

The limiting case in $G^1_{n=3}(D_0)$, when the decision of system (1.1) is representing in the form of (2.10) with conditions (1.2), (1.3), (A_1), (2.1) and

$$
\begin{cases}
\forall(x_1,x_2,x_3,t)\in T;\ f_i;\upsilon_{i0}:\ \sup\limits_{R^3}\left|D^k\upsilon_{i0}\right|\le\beta_1;\quad \sup\limits_{T}\left|D^k\Phi_i\right|\le\gamma_1,(i=\overline{1,3};k=\overline{0,3}),\\
\sup\limits_{T}\dfrac{1}{\sqrt{\pi^3}}\int\limits_0^t\int\limits_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2)\left|D^k\Phi_i(l_1,l_2,l_3;s)\right|d\tau_1 d\tau_2 d\tau_3 ds\le\gamma_1 T_0=\beta_2,
\end{cases}
$$

$$\begin{cases} \sup_T \frac{1}{\sqrt{\pi^3}} \int_0^t \int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)) \frac{1}{\sqrt{t-s}} \sum_{j=1}^3 \left|\tau_j\right| \times \left|\Phi_{il_j}(l_1,l_2,l_3;s)\right| d\tau_1 d\tau_2 d\tau_3 ds \le 3\gamma_1\sqrt{2T_0} = \beta_3, \\ \sup_{R^3} \int_0^{T_0} \left|\Phi_i(x_1,x_2,x_3,s)\right| ds \le \gamma_1 T_0 = \beta_2, \quad (i=\overline{1,3}), \\ \sup_{R^3} \frac{1}{\sqrt{\pi^3}} \int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2))(\sum_{i=1}^3 \left|\tau_j\right| \times \left|\upsilon_{i0\bar{l}_j}(\bar{l}_1,\bar{l}_2,\bar{l}_3)\right|) d\tau_1 d\tau_2 d\tau_3 \le \beta_1 \frac{1}{\sqrt{\pi^3}} \times \\ \times \{\sum_{i=1}^3 (\int_{R^3} \tau_i^2 \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)) d\tau_1 d\tau_2 d\tau_3)^{\frac{1}{2}} (\int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)) d\tau_1 d\tau_2 d\tau_3)^{\frac{1}{2}}\} = \\ = 3\beta_1 \frac{1}{\sqrt{2}}, \quad (l_i = x_i + 2\tau_i\sqrt{\mu(t-s)}; \ \bar{l}_i = x_i + 2\tau_i\sqrt{\mu t}; \ i=\overline{1,3}), \\ \frac{1}{\sqrt{\pi^3}} \int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)) d\tau_1 d\tau_2 d\tau_3 = 1; \quad \beta = \max_{1\le i\le 3} \beta_i; \quad \beta_0 = \beta(3\sqrt{2\mu T_0} + 1 + T_0\sqrt{\mu}). \end{cases} \tag{2.12}$$

Really, estimating (2.10) in $G^1_{n=3}(D_0)$, we have

$$\begin{cases} \|v\|_{G^1_{n=3}(D_0)} = \sum_{i=1}^3 [\|\upsilon_i\|_{C^{3,0}(T)} + \|\upsilon_{it}\|_{L^1}] \le 3[N_1 + \beta_0] = M^*, \\ \|\upsilon_i\|_{C^{3,0}(T)} = \sum_{0\le|k|\le 3} \|D^k\upsilon_i\|_{C(T)} \le N_1 = 40\beta, \quad (\|\upsilon_i\|_{C(T)} \le 2\beta; \ i=\overline{1,3}), \\ \|\upsilon_{it}\|_{L^1} = \sup_{R^3} \int_0^{T_0} \left|\upsilon_{it}(x_1,x_2,x_3,t)\right| dt \le \beta(3\sqrt{2\mu T_0} + 1 + T_0\sqrt{\mu}) = \beta_0, (i=\overline{1,3}), \\ \left|\upsilon_i\right| = \frac{1}{\sqrt{\pi^3}} \sup_T \int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)) \left|\upsilon_{i0}(x_1+2\tau_1\sqrt{\mu t}, x_2+2\tau_2\sqrt{\mu t}, x_3+2\tau_3\sqrt{\mu t})\right| d\tau_1 d\tau_2 d\tau_3 + \\ + \frac{1}{\sqrt{\pi^3}} \sup_T \int_0^t \int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)) \left|\Phi_i(x_1+2\tau_1\sqrt{\mu(t-s)}, x_2+2\tau_2\sqrt{\mu(t-s)}, x_3+2\tau_3 \times \right. \\ \left. \times\sqrt{\mu(t-s)}; s)\right| d\tau_1 d\tau_2 d\tau_3 ds \le \beta_1 + \beta_2 \le 2\beta, \forall (x_1,x_2,x_3,t) \in T, (i=\overline{1,3}). \end{cases}$$

The singleness of the solution $\upsilon_i \in C^{3,0}(T)$ the system (2.10) is obvious on the basis of proof by contradiction [13]. Results (2.10) with a condition ((A_1), (2.1)) are received where smoothness of functions is required only on x_i as the derivative of 1st order is in time has $t>0$.

Remark 1. Alternatively, we can consider, e.g., a class of suitable solutions constructed in $W^2_\lambda(D_0)$.

Let the decision of system (1.1) is representing in the form of (2.10) with conditions (1.2), (1.3), (A_1), (2.12) and

$$\begin{cases} \forall (x_1,x_2,x_3,t) \in T: \ \sup_T \left|D^k\Phi_i\right| \le \gamma_1, \quad (i=\overline{1,3}; k=\overline{0,3}), \\ (\sup_{R^3} \int_0^{T_0} \lambda(s)\left|\Phi_i(x_1,x_2,x_3,s)\right|^2 ds)^{\frac{1}{2}} \le \gamma_1\sqrt{q_1} = \beta_4, \ [0 \le \lambda(t): \int_0^{T_0} \lambda(t)\frac{1}{t}dt = q_0; \ \int_0^{T_0} \lambda(t)dt = q_1], \\ \beta_* = \max(\beta,\beta_4); \quad \tilde{\beta}_0 = \beta_*(3\sqrt{\mu q_0} + 1 + \sqrt{\mu q_1}), \end{cases} \tag{2.13}$$

that decision (2.10) of problem Navier-Stokes (1.1) - (1.3) belongs in $W_\lambda^2(D_0)$.

Really, estimating (2.10) in $W_\lambda^2(D_0)$, we have [8]:

$$\begin{cases} \|v\|_{W_\lambda^2} = \sum_{i=1}^{3} \|\upsilon_i\|_{\tilde{W}^2_{(\upsilon_i,\lambda)}} \le 3[N_1\sqrt{T_0} + \tilde{\beta}_0] = 3M^*, \\ \|\upsilon_i\|_{\tilde{W}^2_{(\upsilon_i,\lambda)}} \le N_1\sqrt{T_0} + \tilde{\beta}_0 = M^*, \quad (N_1 = 40\beta;\ i = \overline{1,3}), \\ (D_0 = R^3 \times (0,T_0), v = (\upsilon_1,\upsilon_2,\upsilon_3)), \\ \|\upsilon_{it}\|_{L_\lambda^2} = (\sup_{R^3} \int_0^{T_0} \lambda(t)|\upsilon_{it}(x_1,x_2,x_3,t)|^2 dt)^{\frac{1}{2}} \le \beta_*(3\sqrt{\mu q_0} + 1 + \sqrt{\mu q_1}) = \tilde{\beta}_0, (i = \overline{1,3}), \end{cases}$$

i.e. in the conditions of (1.2), (1.3), (A_1) and (2.12), (2.13) the problem (1.1) - (1.3) has a limited solution in $W_\lambda^2(D_0)$. Let's notice that in work [8] similar results in case of (2.10) also are received in the weight space $G_\lambda^2(D_0)$.

3) The essence of this subparagraph to define the decision (2.9) in $\tilde{C}^{3,1}_{n=3}(T)$. For this purpose of problem (2.9), (1.3) it is possible to solve differently if conditions are satisfied:

$$\begin{cases} \Delta\upsilon_{i0} = 0;\ \upsilon_{i0} \in C^3(R^3); \Phi_i:\ \sup_{R^3}|D^k\upsilon_{i0}| \le \beta_1;\ \sup_T |D^k\Phi_i(x_1,x_2,x_3,t)| \le \gamma_1,\ (i = \overline{1,3}), \\ \sup_T \frac{1}{\sqrt{\pi^3}} \int_0^t \int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)) |D^k\Phi_i(l_1,l_2,l_3;s)| d\tau_1 d\tau_2 d\tau_3 ds \le \gamma_1 T_0 = \gamma_2, (i = \overline{1,3}), \\ \sup_T \frac{1}{\sqrt{\pi^3}} \int_0^t \int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)) \frac{1}{\sqrt{t-s}} \sum_{j=1}^{3} |\tau_j| \times |\Phi^{(k)}_{il_j^k}(l_1,l_2,l_3;s)| d\tau_1 d\tau_2 d\tau_3 ds \le \\ \le \gamma_1 \frac{1}{\sqrt{\pi^3}} \sup_T \int_0^t \frac{1}{\sqrt{t-s}} \{\sum_{j=1}^{3} (\int_{R^3} \tau_j^2 \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)) d\tau_1 d\tau_2 d\tau_3)^{\frac{1}{2}} \times \\ \times (\int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)) d\tau_1 d\tau_2 d\tau_3)^{\frac{1}{2}}\} ds = 3\gamma_1 \frac{1}{\sqrt{2}} \sup_{[0,T_0]} \int_0^t \frac{1}{\sqrt{t-s}} ds = 3\gamma_1\sqrt{2T_0} = \gamma_3, \\ l_j = x_j + 2\tau_j\sqrt{\mu(t-s)}, (j = \overline{1,3};\ k = \overline{0,3}),\ \gamma = \max_{1\le i\le 3}(\gamma_i;\beta_1),\ \gamma_0 = \gamma(1+\sqrt{\mu}). \end{cases} \tag{2.12*}$$

Then speed components v are defined by a rule

$$\begin{cases} \upsilon_i = \upsilon_{i0}(x_1,x_2,x_3) + V_i(x_1,x_2,x_3,t), \forall(x_1,x_2,x_3,t) \in T, (i = \overline{1,3}), \\ V_i|_{t=0} = 0,\ \forall(x_1,x_2,x_3) \in R^3. \end{cases} \tag{2.14}$$

Then the system (2.9) will be transformed referring to

$$V_{it} = \Phi_i + \mu\Delta V_i, (i = \overline{1,3}). \tag{2.15}$$

Where V_i new unknown functions which defines the decision of problem Navier-Stokes. Hence

$$V_i = \frac{1}{8\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-\frac{r^2}{4\mu(t-s)})\Phi_i(s_1,s_2,s_3,s)\frac{ds_1ds_2ds_3ds}{\sqrt{(\mu(t-s))^3}} = \frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\times$$
$$\times\Phi_i(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)d\tau_1d\tau_2d\tau_3ds \equiv \quad (2.16)$$
$$\equiv \overline{H}_i(x_1,x_2,x_3,t),\ (s_i - x_i = 2\tau_i\sqrt{\mu(t-s)}; i=\overline{1,3}).$$

The found decision (2.16) satisfies system (2.15). Really, having calculated partial derivative of system (2.16):

$$\begin{cases} (0,1)\ni\mu;\ \forall(x_1,x_2,x_3,t)\in T: \\ V_{it} = \Phi_i(x_1,x_2,x_3,t) + \frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\sum_{j=1}^{3}\sqrt{\mu}\frac{\tau_j}{\sqrt{t-s}}\Phi_{il_j}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+ \\ +2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)d\tau_1d\tau_2d\tau_3ds,\ (l_j = x_j+2\tau_j\sqrt{\mu(t-s)};\ \ i=\overline{1,3};\ j=\overline{1,3}), \\ V_{ix_j} = \frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Phi_{il_j}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\times \\ \times\sqrt{\mu(t-s)};s)d\tau_1d\tau_2d\tau_3ds, \\ V_{ix_j^2} = \frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Phi_{il_j^2}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\times \\ \times\sqrt{\mu(t-s)};s)d\tau_1d\tau_2d\tau_3ds, \end{cases} \quad (2.17)$$

and substituting (2.17) in (2.15), we have (see (*)):

$$\begin{cases} V_i\big|_{t=0} = 0,\ \forall(x_1,x_2,x_3)\in R^3;\ \ (0,1)\ni\mu;\ \forall(x_1,x_2,x_3,t)\in T: \\ 0 = V_{it} - \Phi_i - \mu\Delta V_i \equiv \Phi_i + \frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\times(\sum_{j=1}^{3}\sqrt{\mu}\frac{\tau_j}{\sqrt{t-s}}\Phi_{il_j}(x_1+ \\ +2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s))d\tau_1d\tau_2d\tau_3ds - \Phi_i - \\ -\mu\{\frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\times(\sum_{j=1}^{3}\Phi_{il_j^2}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)}, \\ x_3+2\tau_3\sqrt{\mu(t-s)};s))d\tau_1d\tau_2d\tau_3ds\} = \frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\times(\sum_{j=1}^{3}\sqrt{\mu}\frac{\tau_j}{\sqrt{t-s}}\times \\ \times\Phi_{il_j}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s))d\tau_1d\tau_2d\tau_3ds - \\ -\frac{1}{2}\sqrt{\mu}\{\frac{1}{\sqrt{\pi^3}}\int_0^t\frac{1}{\sqrt{t-s}}[\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Phi_{il_1^2}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)}, \\ x_3+2\tau_3\sqrt{\mu(t-s)};s)d(x_1+2\tau_1\sqrt{\mu(t-s)})d\tau_2d\tau_3 + \int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Phi_{il_2^2}(x_1+ \\ +2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)d\tau_1d(x_2+2\tau_2\sqrt{\mu(t-s)})d\tau_3 + \\ +\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Phi_{il_3^2}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)\times \end{cases}$$

$$\begin{cases} \times d\tau_1 d\tau_2 d(x_3+2\tau_3\sqrt{\mu(t-s)})]ds\} = \frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\times(\sum_{j=1}^3\sqrt{\mu}\frac{\tau_j}{\sqrt{t-s}}\Phi_{il_j}(x_1+ \\ +2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s))d\tau_1 d\tau_2 d\tau_3 ds - \\ -\sqrt{\mu}\{\frac{1}{\sqrt{\pi^3}}\int_0^t\frac{1}{\sqrt{t-s}}\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\times(\sum_{j=1}^3\tau_j\Phi_{il_j}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)}, \\ x_3+2\tau_3\sqrt{\mu(t-s)};s))d\tau_1 d\tau_2 d\tau_3 ds = 0. \end{cases} \quad (2.18)$$

That it was required to prove.

Therefore on the basis of (2.14), (2.16) we will receive

$$\begin{cases} \upsilon_i = \upsilon_{i0} + \overline{H}_i \equiv \upsilon_{i0} + \frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Phi_i(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)}, \\ x_3+2\tau_3\sqrt{\mu(t-s)};s)d\tau_1 d\tau_2 d\tau_3 ds \equiv H_i,(i=\overline{1,3}), \\ \Delta\upsilon_{i0}=0;\quad \upsilon_{i0}\in C^3(R^3);\quad \sum_{i=1}^3\upsilon_{x_i} = \sum_{i=1}^3 H_{ix_i} = 0,(i=\overline{1,3}). \end{cases} \quad (2.19)$$

Limitation of functions $(\upsilon_1,\upsilon_2,\upsilon_3)$ in $\tilde{C}^{3,1}_{n=3}(T)$. The limiting case which we will consider concern results of the theorem 1. Then the decision of system (1.1) is representing in the form of (2.19) with conditions (1.2), (1.3), (A_1), (2.1) and (2.12)*.

Really, estimating (2.19) in $\tilde{C}^{3,1}_{n=3}(T)$, we have

$$\begin{cases} \|v\|_{\tilde{C}^{3,1}_{n=3}(T)} = \sum_{i=1}^3\{\|\upsilon_i\|_{C^{3,0}(T)} + \|\upsilon_{it}\|_{C(T)}\} \le 3[N_1+\gamma_0] = M^*, \\ \|\upsilon_i\|_{C^{3,0}(T)} = \sum_{0\le|k|\le 3}\|D^k\upsilon_i\|_{C(T)} \le N_1 = 40\gamma,(\|\upsilon_i\|_{C(T)} \le 2\gamma; i=\overline{1,3}), \\ \|\upsilon_{it}\|_{C(T)} \le \gamma_0 = \gamma(1+\sqrt{\mu}),(i=\overline{1,3}), \end{cases} \quad (2.20)$$

so as

$$\begin{cases} |\upsilon_i| = |\upsilon_{i0}| + |\overline{H}_i| \le \beta_1+\gamma_2 \le 2\gamma,(\max(\beta_1;\gamma_2)\le\gamma), \\ |\overline{H}_i| \le \frac{1}{\sqrt{\pi^3}}\sup_T\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))|\Phi_i(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\times \\ \times\sqrt{\mu(t-s)};s)|d\tau_1 d\tau_2 d\tau_3 ds \le \gamma_2,\forall(x_1,x_2,x_3,t)\in T,(i=\overline{1,3}). \end{cases}$$

Singleness is obvious, as a method by contradiction. Therefore from (2.19) singleness of the solution follows in $\tilde{C}^{3,1}_{n=3}(T)$.

4) Coming back to the proof the theorem 1, thus considering (2.3), (2.19), and their partial derivatives on x_i, we find

$$\theta_{x_i} = \sum_{j=1}^3(H_j\cdot H_{ix_j} - H_j\cdot H_{jx_i}) \equiv \psi_i(x_1,x_2,x_3,t), i=\overline{1,3}. \quad (2.21)$$

As ψ_i – is known functions, hence from system (2.21) differentiating 1-equation on x_1 [(2.21): $i=1$], 2- equations on x_2 [(2.21): $i=2$], 3-equations on x_3 [(2.21): $i=3$], summing up, we will receive

$$\Delta\theta = -\psi^0, (\psi^0 \equiv -\sum_{i=1}^{3} \psi_{ix_i}(x_1, x_2, x_3, t)), \tag{2.22}$$

at that

$$\theta \in C^2(T): \quad \theta = \frac{1}{4\pi}\int_{R^3} \psi^0(s_1, s_2, s_3, t)\frac{ds_1 ds_2 ds_3}{r}.$$

The equation (2.22) it have the third equation of system (2.4). Therefore, from the received results, taking into account (2.6), follows

$$\frac{1}{\rho}P + \frac{1}{2}Q = -\theta + \frac{1}{4\pi}\int_{R^3} F_0(s_1, s_2, s_3, t)\frac{ds_1 ds_2 ds_3}{r}, \tag{2.23}$$

i.e. (2.23) – is the fourth equation of system (2.4).

The formula (2.23) can be transformed in equivalent form

$$\begin{cases} \frac{1}{\rho}P + \frac{1}{2}\sum_{i=1}^{3}\upsilon_i^2 = \int_{R^3}\frac{1}{r}\Upsilon(s_1, s_2, s_3, t)ds_1 ds_2 ds_3, \\ \theta = \frac{1}{4\pi}\int_{R^3}\psi^0(s_1, s_2, s_3, t)\frac{ds_1 ds_2 ds_3}{r}, \quad (Q \equiv \sum_{i=1}^{3}\upsilon_i^2; \ \Upsilon \equiv \frac{1}{4\pi}(F_0 - \psi^0)), \end{cases} \tag{2.23*}$$

where (2.23)* – the equation of Bernoulli's type [12]. Then function (2.23)*: $\frac{1}{\rho}P + \frac{1}{2}\sum_{i=1}^{3}\upsilon_i^2 \equiv \tilde{I}$

satisfies the equation:

$$\Delta\tilde{I} = -4\pi\Upsilon,$$

and function [13]: $\tilde{I}$ is called Newton's potential, at that on infinity aspires to a zero. Υ – is called density of this potential.

Hence functions υ_i, θ, P are defined from systems (2.19), (2.22), (2.23) and these functions are smooth on set to the variables, that the system (2.4) has the single smooth solution. The theorem 1 – is proved. ■

As a consequence the theorem 1 we will receive following statements:

Theorem 2. In conditions of the theorem 1 and (2.10), (2.12) problem Navier-Stokes (1.1)-(1.3), (A_1) is solvable at $G^1_{n=3}(D_0)$.

Theorem 2*. In conditions of the theorem 1 and (2.12)*, (2.20) the problem (1.1) - (1.3), (A_1) has the smooth single solution in $\tilde{C}^{3,1}_{n=3}(T)$.

The essential factor of researches of this paragraph are results of the theorem 2*. In this case the decision of system (1.1) is considered as the strict decision of a problem (1.1) - (1.3), (A_1).

It is obvious that small changes $\upsilon_{i0},(i=\overline{1,3})$ or $f_i,(i=\overline{1,3})$ influence the decision (2.19) a little, i.e. continuous depends on this data. Therefore, a question on a statement correctness problems (1.1)-(1.3), (A_1) are considered at once with results of the theorem 2*.

2.2. Inequality Beale-Kato-Majda

The Beale-Kato-Majda regularity criterion originally derived for solutions to the 3D Euler equations [2] and holds for solutions to the 3D equations Navier-Stokes [5] and the criterion can be viewed as a continuation principle for strong solutions. A further generalization was presented in [8] where the regularity condition is expressed in terms of the time integrability.

Note that there are some inequalities for a priori estimates depending on the spaces. To prove this criterion, for example, enough fulfill the inequality [5]:

$$\sup_{R^3}\int_0^{T_0}\left|\mathrm{rot}\,v(x_1,x_2,x_3,t)\right|dt \le M = \mathrm{const} < \infty. \tag{2.24}$$

On the basis of results of the theorem 1 the solution of systems (1.1) it is presented in a kind (2.19), where global existence of decisions is received in a class $\tilde{C}^{3,1}_{n=3}(T)$ from the point of view of the initial data satisfying (2.19). It is pleasant that results of this theorem leads to such global classical solution Navier-Stokes, besides it is known that in [5] classical solution is received, if the criterion of Beale-Kato-Majda is executed.

Really, at performance of conditions of the theorem 2* takes place

$$\begin{cases}
\left|\dfrac{\partial}{\partial x_2}\upsilon_{30}(x_1,x_2,x_3)-\dfrac{\partial}{\partial x_3}\upsilon_{20}(x_1,x_2,x_3)\right|\le h_1^0; \quad \left|\dfrac{\partial}{\partial x_3}\upsilon_{10}(x_1,x_2,x_3)-\dfrac{\partial}{\partial x_1}\upsilon_{30}(x_1,x_2,x_3)\right|\le h_2^0,\\
\left|\dfrac{\partial}{\partial x_1}\upsilon_{20}(x_1,x_2,x_3)-\dfrac{\partial}{\partial x_2}\upsilon_{10}(x_1,x_2,x_3)\right|\le h_3^0; \quad \sup\limits_T\dfrac{1}{\sqrt{\pi}^3}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\left|\dfrac{\partial}{\partial x_2}\Phi_3(x_1+\right.\\
+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)-\dfrac{\partial}{\partial x_3}\Phi_2(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+\\
\left.+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)\right|d\tau_1d\tau_2d\tau_3ds\le h_1,\\
\sup\limits_T\dfrac{1}{\sqrt{\pi}^3}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\left|\dfrac{\partial}{\partial x_3}\Phi_1(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\times\right.\\
\left.\times\sqrt{\mu(t-s)};s)-\dfrac{\partial}{\partial x_1}\Phi_3(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)\right|\times\\
\times d\tau_1d\tau_2d\tau_3ds\le h_2,\\
\sup\limits_T\dfrac{1}{\sqrt{\pi}^3}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\left|\dfrac{\partial}{\partial x_1}\Phi_2(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\times\right.\\
\left.\times\sqrt{\mu(t-s)};s)-\dfrac{\partial}{\partial x_2}\Phi_1(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)\right|\times\\
\times d\tau_1d\tau_2d\tau_3ds\le h_3, \quad (\sum\limits_{i=1}^{3}(h_i+h_i^0)=M_0=\mathrm{const}<\infty).
\end{cases}$$

Then we will receive estimation

$$\mathrm{rot}\,\nu \neq 0: \quad \sup_{T} \left|\mathrm{rot}\,\nu(x_1, x_2, x_3, t)\right| \leq M_0 < \infty .$$

In a consequence and (see (2.24)):

$$\sup_{R^3} \int_0^{T_0} \left|\mathrm{rot}\,\nu(x_1,x_2,x_3,t)\right| dt \leq \sup_{R^3} \int_0^{T_0} \{ \left| \frac{\partial}{\partial x_2}\upsilon_3(x_1,x_2,x_3,s) - \frac{\partial}{\partial x_3}\upsilon_2(x_1,x_2,x_3,s)\right| + \left| \frac{\partial}{\partial x_3}\upsilon_1(x_1,x_2,x_3,s) - \right.$$

$$\left. - \frac{\partial}{\partial x_1}\upsilon_3(x_1,x_2,x_3,s)\right| + \left| \frac{\partial}{\partial x_1}\upsilon_2(x_1,x_2,x_3,s) - \frac{\partial}{\partial x_2}\upsilon_1(x_1,x_2,x_3,s)\right| \} ds \leq M_0 T_0 = M < \infty .$$

2.3. Estimation of affinity of decisions of the equations Navier-Stokes and Euler

I. Incompressible Streams Without the Friction. For incompressible currents without a friction [2, 9 and 12]: $\mu = 0$ the equations of Navier-Stokes become simpler, as there are no members: $\Delta \upsilon_i$. Therefore the problem (1.1) - (1.3) is led to a kind

$$\bar{\upsilon}_{it} + \frac{1}{2}\Big(\sum_{j=1}^{3} \bar{\upsilon}_j^2\Big)_{x_i} = \bar{f}_i - \frac{1}{\rho}\bar{P}_{x_i}, i = \overline{1,3}, \tag{2.25}$$

$$\bar{\upsilon}_i(x_1,x_2,x_3,t)|_{t=0} = \bar{\upsilon}_{0i}(x_1,x_2,x_3), i = \overline{1,3}, \tag{2.26}$$

$$\mathrm{div}\,\bar{\nu} = 0, \bar{\nu} = (\bar{\upsilon}_1, \bar{\upsilon}_2, \bar{\upsilon}_3); \mathrm{rot}\,\bar{\nu} = 0. \tag{2.27}$$

The system (2.25) with conditions (2.26), (2.27) has the strict decision [12] with preservation of all convective members at performance of a condition of Stokes. Really, for incompressible currents without a friction, the vector of speed is represented as a gradient of potential $\bar{\nu}$ and this potential satisfies the Laplace equation.

Therefore for potential currents the member in the equation (1.1), depending on viscosity, identically disappears. At that the system (2.25) has the smooth single solution in $\tilde{C}^{3,1}_{n=3}(T)$ [9, see pp.147-151]:

$$\begin{cases} \|\bar{\nu}\|_{\tilde{C}^{3,1}_{n=3}(T)} = \sum_{i=1}^{3} \|\bar{\upsilon}_i\|_{\tilde{C}^{3,1}(T)} \leq N_0; \quad \|\bar{\upsilon}_i\|_{\tilde{C}^{3,1}(T)} = \sum_{0 \leq |k| \leq 3} \|D^k \bar{\upsilon}_i\|_{C(T)} + \|\bar{\upsilon}_{it}\|_{C(T)}, \\ k = 0: D^0 \bar{\upsilon}_i \equiv \bar{\upsilon}_i; k \neq 0: D^k \bar{\upsilon}_i = \dfrac{\partial^{|k|}\bar{\upsilon}_i}{\partial x_1^{\alpha_1}\partial x_2^{\alpha_2}\partial x_3^{\alpha_3}}; |k| = \sum_{i=1}^{3} \alpha_i, (\alpha_i = 0,1,2,3; i = \overline{1,3}), \end{cases} \tag{2.28}$$

and is harmonious functions.

The specified method of theorem 2* can be used in particular and for the decision of a problem (2.25) - (2.27) as a test example. Really, on a basis APS from system (2.25), follows

$$\begin{cases} \Delta \bar{J} = -\bar{F}_0, \\ \bar{J} \equiv \dfrac{1}{\rho}\bar{P} + \dfrac{1}{2}\bar{Q}, \ \bar{F}_0 \equiv -\sum_{i=1}^{3} \bar{f}_{ix_i}, \end{cases} \tag{2.29}$$

and

$$\begin{cases} \bar{J} = \dfrac{1}{4\pi}\int\limits_{R^3} \bar{F}_0(s_1,s_2,s_3;t)\dfrac{ds_1ds_2ds_3}{r}, \\ \bar{J}_{x_i} = \dfrac{1}{4\pi}\int\limits_{R^3} \bar{F}_0(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t)\dfrac{\tau_i d\tau_1 d\tau_2 d\tau_3}{\sqrt{(\tau_1^2+\tau_2^2+\tau_3^2)^3}}, (i=\overline{1,3}). \end{cases} \quad (2.30)$$

So as $\bar{J}$ - the decision of the equation (2.29), with the account: $\bar{J}_{x_i} \equiv \frac{1}{\rho}\bar{P}_{x_i} + \frac{1}{2}\bar{Q}_{x_i}$ from (2.25) we will receive

$$\bar{\upsilon}_{it} = \bar{f}_i - \bar{J}_{x_i}, i=\overline{1,3}. \quad (2.31)$$

From system (2.31) on the basis of (2.26), follows

$$\begin{cases} \bar{\upsilon}_i = \bar{\upsilon}_{i0}(x_1,x_2,x_3) + \int\limits_0^t \bar{\Phi}_i(x_1,x_2,x_3,\tau)d\tau, (i=\overline{1,3}), \\ \bar{\Phi}_i \equiv \bar{f}_i - \bar{J}_{x_i}, (i=\overline{1,3}), \quad \sum\limits_{i=1}^{3}\bar{\Phi}_{ix_i} \equiv \bar{F}_0 + \Delta\bar{J} = 0, \ \forall(x_1,x_2,x_3,t)\in T. \end{cases} \quad (2.32)$$

Hence from (2.29) and (2.30) we have

$$\frac{1}{\rho}\bar{P} = -\frac{1}{2}\bar{Q} + \frac{1}{4\pi}\int\limits_{R^3}\bar{F}_0(s_1,s_2,s_3,t)\frac{ds_1ds_2ds_3}{r}, \quad (2.33)$$

i.e. (2.33) there is an equation of type Bernoulli that is similar with (2.23)*.

From the received results follows that the system (2.32) satisfies to a condition (2.27), and it means that the found decisions $\bar{\upsilon}_i, i=\overline{1,3}$ satisfies to the of Laplace equation, i.e. are harmonious functions. As it has been proved.

II. It is known that limit transition to very small viscosity should be executed not in the equations of Navier-Stokes, but in the decision of these equations by approach of factor of viscosity to zero [12]. Then the solution of system (1.1) is representing in the form of (2.19) with conditions of theorem 2*.

To estimate affinity of decisions (2.19), (2.32) in sense $\tilde{C}^{3,1}_{n=3}(T)$, when [9]: $\mu \to 0$, conditions are required is:

$$\begin{cases} \forall(x_1,x_2,x_3,t)\in T; \Phi_i, \bar{\Phi}_i, \upsilon_{i0} \in C^3(R^3): \ \left|D^k(\upsilon_{i0} - \bar{\upsilon}_{i0})\right| < \lambda_1\delta_{1\mu}, \forall(x_1,x_2,x_3)\in R^3, (i=\overline{1,3}), \\ \left|D^k[\Phi_i(x_1,x_2,x_3,t) - \bar{\Phi}_i(x_1,x_2,x_3,t)]\right| \le \lambda_2\delta_{2\mu}, \forall(x_1,x_2,x_3,t)\in T, (i=\overline{1,3}), \\ \dfrac{1}{\sqrt{\pi^3}}\sup\limits_T \int\limits_0^t\int\limits_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\{\left|D^k[\Phi_i(x_1+2\tau_1\sqrt{\mu(t-s)}, x_2+2\tau_2\sqrt{\mu(t-s)}, x_3 + \right. \\ \left. +2\tau_3\sqrt{\mu(t-s)};s) - \Phi_i(x_1,x_2,x_3;s)]\right|\}d\tau_1d\tau_2d\tau_3ds < \lambda_3\sqrt{\mu}, (i=\overline{1,3}), \\ \sqrt{\mu}\dfrac{1}{\sqrt{\pi^3}}\sup\limits_T\int\limits_0^t\int\limits_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))[\sum\limits_{j=1}^{3}\dfrac{|\tau_j|}{\sqrt{t-s}}\left| \ \Phi_{il_j}(x_1+2\tau_1\sqrt{\mu(t-s)}, x_2+2\tau_2\sqrt{\mu(t-s)}, x_3 + \right. \\ \left. +2\tau_3\sqrt{\mu(t-s)};s)\right|]d\tau_1d\tau_2d\tau_3ds < \lambda_4\sqrt{\mu}, \quad (l_i = x_i + 2\tau_i\sqrt{\mu(t-s)}; \ i=\overline{1,3}), \\ 0 < \lambda_k \le \lambda_0 = \text{const} < \infty; \ k=\overline{1,4}; \ \delta_{1\mu} = \delta_{2\mu} = \delta = \sqrt{\mu}. \end{cases} \quad (2.34)$$

Lemma 1. If conditions of the theorem 2* and (2.34) are satisfied, an admissible error between decisions of system (2.19), (2.32) in $\tilde{C}^{3,1}_{n=3}(T)$, when $\delta=\sqrt{\mu}$, will be an order $O(\sqrt{\mu})$.

Proof. To prove to affinity of decisions (2.19) and (2.32) in $\tilde{C}^{3,1}_{n=3}(T)$, at first we will prove to affinity of decisions $C(T), C^{3,0}(T)$. At that obviously that estimations relatively $\|\upsilon_i-\bar{\upsilon}_i\|_{\tilde{C}^{3,1}(T)}$ will be an order $O(\sqrt{\mu})$.

Really estimating (2.19), (2.32), we have

$$|\upsilon_i-\bar{\upsilon}_i|\le|\upsilon_{i0}-\bar{\upsilon}_{i0}|+\frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))|\Phi_i(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},$$

$$x_3+2\tau_3\sqrt{\mu(t-s)};s)-\Phi_i(x_1,x_2,x_3;s)|d\tau_1 d\tau_2 d\tau_3 ds+\frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\times$$

$$\times|\Phi_i(x_1,x_2,x_3;s)-\bar{\Phi}_i(x_1,x_2,x_3;s)|]d\tau_1 d\tau_2 d\tau_3 ds\le\lambda_1\delta_{1\mu}+\lambda_3\sqrt{\mu}+\lambda_2\delta_{2\mu}T_0=C_0\sqrt{\mu},$$

$$C_0=\lambda_1+\lambda_3+\lambda_2 T_0,\ (\delta_{1\mu}=\delta_{2\mu}=\sqrt{\mu};\ i=\overline{1,3}),$$

or

$$\|\upsilon_i-\bar{\upsilon}_i\|_{C(T)}\le C_0\sqrt{\mu},(i=\overline{1,3}),$$

here (see.(2.32)):

$$\bar{\upsilon}_i=\bar{\upsilon}_{i0}(x_1,x_2,x_3)+\int_0^t\bar{\Phi}_i(x_1,x_2,x_3,\tau)d\tau=\bar{\upsilon}_{i0}+\frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\times$$

$$\times\bar{\Phi}_i(x_1,x_2,x_3,\tau)d\tau_1 d\tau_2 d\tau_3 d\tau\equiv\bar{H}_i,(i=\overline{1,3};\quad \frac{1}{\sqrt{\pi^3}}\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))d\tau_1 d\tau_2 d\tau_3=1). \tag{2.32*}$$

Similarly, we will receive also estimations concerning expressions where partial derivative functions υ_i and $\bar{\upsilon}_i$ to the third order, inclusive, contain υ_i, $\bar{\upsilon}_i$, i.e.

$$\|\upsilon_i-\bar{\upsilon}_i\|_{C^{3,0}(T)}=\sum_{0\le|k|\le3}\|D^k(\upsilon_i-\bar{\upsilon}_i)\|_{C(T)}\le 20C_0\sqrt{\mu},(i=\overline{1,3}).$$

Hence, as $\tilde{C}^{3,1}(T)\ni\bar{\upsilon}_i,\upsilon_i$, estimating (2.19) and (2.32)* in sense of norm $\tilde{C}^{3,1}(T)$ we will receive:

$$\begin{cases}\|\upsilon_i-\bar{\upsilon}_i\|_{\tilde{C}^{3,1}(T)}=\sum_{0\le|k|\le3}\|D^k(\upsilon_i-\bar{\upsilon}_i)\|_{C(T)}+\|\upsilon_{it}-\bar{\upsilon}_{it}\|_{C(T)}\le(20C_0+2\lambda_0)\sqrt{\mu}=N_0\sqrt{\mu},(i=\overline{1,3}),\\ \|\upsilon_{it}-\bar{\upsilon}_{it}\|_{C(T)}\le 2\lambda_0\sqrt{\mu},(i=\overline{1,3}),\\ \bar{\upsilon}_{it}=\bar{\Phi}_i,\ (i=\overline{1,3});\ \upsilon_{it}=\frac{\partial}{\partial t}[\upsilon_{i0}(x_1,x_2,x_3)+\bar{H}_i(x_1,x_2,x_3,t)]=\bar{H}_{it},\ (V_{it}\equiv\bar{H}_{it};\quad \text{see.(2.19)}),\\ \bar{H}_{it}=\Phi_i(x_1,x_2,x_3,t)+\frac{1}{\sqrt{\pi^3}}\sqrt{\mu}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))[\sum_{j=1}^3\frac{\tau_j}{\sqrt{t-s}}\Phi_{il_j}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+\\ +2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)]d\tau_1 d\tau_2 d\tau_3 ds,(i=\overline{1,3}).\end{cases} \tag{2.35}$$

Then taking into account (2.35) and $\nu = (\upsilon_1, \upsilon_2, \upsilon_3), \bar{\nu} = (\bar{\upsilon}_1, \bar{\upsilon}_2, \bar{\upsilon}_3)$, we have

$$\|\nu - \bar{\nu}\|_{\tilde{C}^{3,1}_{n=3}(T)} = \sum_{i=1}^{3} \|\upsilon_i - \bar{\upsilon}_i\|_{\tilde{C}^{3,1}(T)} = \sum_{i=1}^{3} \{ \sum_{0 \le |k| \le 3} \|D^k(\upsilon_i - \bar{\upsilon}_i)\|_{C(T)} + \|\upsilon_{it} - \bar{\upsilon}_{it}\|_{C(T)} \} \le 3N_0\sqrt{\mu}, (i = \overline{1,3}). \quad (2.36)$$

And it means that if $\delta = \sqrt{\mu}$ the admissible error of an estimation will be order $O(\sqrt{\mu})$ in $\tilde{C}^{3,1}_{n=3}(T)$. The lemma 1 – is proved. ■

3. Fluid average Viscosity with a Condition (A_2)

Let's consider a fluid with viscosity with Reynolds small number where all inertial participants contain in equations Navier-Stokes. Theoretically, it is not investigated till now [12]. Hence, here we will consider, methods of integration of the equations Navier-Stokes, when: $1 < \mu = \mu_0 = \text{const} < \infty$. Therefore the decision of the method, from where follows of equations integration of Navier-Stokes in a case (A_2), is a major factor of this point. The developed method of the decision of system (1.1) is connected with θ_i, where these functions will transform (1.1) to systems (1.4), (1.5) with conditions (a_{02}) and

$$(1.3): \quad \upsilon_i\big|_{t=0} = 0, \forall (x_1, x_2, x_3) \in R^3, (\upsilon_{i0}(x_1, x_2, x_3) \equiv 0; i = \overline{1,3}), \quad (1.3)^*$$

$$\theta_i\big|_{t=0} = 0, \forall (x_1, x_2, x_3) \in R^3, (i = \overline{1,3}), \quad (3.1)$$

where the current is considered with average size of viscosity.

Theorem 3. Systems (1.4), (1.5) it is equivalent will be transformed to a kind

$$\begin{cases} \Delta J_0 = -F_0, (J_0 \equiv \frac{1}{\rho}P + \frac{1}{2}Q; F_0 = -\sum_{i=1}^{3} f_{ix_i}; \quad \operatorname{div} f \ne 0;\ 1 < \mu = \mu_0 = \text{const} < \infty), \\ \upsilon_{it} = f_i + \mu\Delta\upsilon_i - J_{0x_i} - \theta_i, \\ \theta_i = D_i[\theta_1, \theta_2, \theta_3], i = \overline{1,3}, \\ \frac{1}{\rho}P = -\frac{1}{2}Q + \frac{1}{4\pi}\int_{R^3} F_0(s_1, s_2, s_3, t)\frac{ds_1 ds_2 ds_3}{r}, (r = \sqrt{(x_1 - s_1)^2 + (x_2 - s_2)^2 + (x_3 - s_3)^2}), \end{cases} \quad (3.2)$$

when conditions (1.2), (1.3)*, (3.1), (A_2) are satisfied. Hence, the nonstationary problem of Navier-Stokes (1.1)-(1.3)* has the smooth single solution.

□**Proof.** Really, from system (1.4), considering conditions (1.2), (1.3)*, (3.1) and having entered APS, i.e. differentiating the equations of system (1.4) accordingly on x_i and, then summing up, we have the equation

$$\begin{cases} \Delta J_0 = -F_0, \\ J_0 = \frac{1}{4\pi}\int_{R^3} F_0(s_1, s_2, s_3, t)\frac{ds_1 ds_2 ds_3}{r}. \end{cases} \quad (3.3)$$

If J_0 – the decision of the equation (3.3), then substituting:

$$J_{0x_i} = \frac{1}{4\pi}\int_{R^3}\frac{\tau_i F_0(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t)}{\sqrt{(\tau_1^{\,2}+\tau_2^{\,2}+\tau_3^{\,2})^3}}d\tau_1 d\tau_2 d\tau_3,(s_i - x_i = \tau_i; i=\overline{1,3};\ \ \frac{1}{\rho}P_{x_i} + \frac{1}{2}Q_{x_i} \equiv J_{0x_i})$$

in system (1.4), we have

$$\begin{cases} \upsilon_{it} = \Phi_i - \theta_i + \mu\Delta\upsilon_i, (i=\overline{1,3}), \\ \Phi_i(x_1,x_2,x_3,t) \equiv f_i - J_{0x_i}, (i=\overline{1,3}),\ \sum_{i=1}^{3}\Phi_{ix_i} \equiv F_0 + \Delta J_0 = 0,\ \forall(x_1,x_2,x_3,t)\in T. \end{cases} \tag{3.4}$$

The decision of a problem (1.1) - (1.3)* is represented in a kind

$$\begin{cases} \upsilon_i = H_i^0 - \frac{1}{8\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-\frac{r^2}{4\mu(t-\tau)})\frac{1}{(\sqrt{\mu(t-\tau)})^3}\theta_i(s_1,s_2,s_3,\tau)ds_1ds_2ds_3d\tau \equiv \Phi_i\theta_i, (i=\overline{1,3}), \\ H_i^0(x_1,x_2,x_3,t) \equiv \frac{1}{8\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-\frac{r^2}{4\mu(t-\tau)})\frac{1}{(\sqrt{\mu(t-\tau)})^3}\Phi_i(s_1,s_2,s_3,\tau)ds_1ds_2ds_3d\tau, (i=\overline{1,3}), \end{cases} \tag{3.5}$$

where concerning functions $\theta_i, (i=\overline{1,3})$, we will receive

$$\begin{aligned} \theta_i = \sum_{j=1}^{3}\{[H_j^0 - \frac{1}{\sqrt{\pi}^3}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\theta_j(x_1+2\tau_1\sqrt{\mu(t-\tau)}, x_2+2\tau_2\sqrt{\mu(t-\tau)}, x_3 + \\ +2\tau_3\sqrt{\mu(t-\tau)};\tau)d\tau_1d\tau_2d\tau_3d\tau]\times[H_{ix_j}^0 - \frac{1}{\sqrt{\pi}^3}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\frac{\tau_j}{\sqrt{\mu(t-\tau)}}\theta_i(x_1 + \\ +2\tau_1\sqrt{\mu(t-\tau)}, x_2+2\tau_2\sqrt{\mu(t-\tau)}, x_3+2\tau_3\sqrt{\mu(t-\tau)};\tau)d\tau_1d\tau_2d\tau_3d\tau] - [H_j^0 - \\ - \frac{1}{\sqrt{\pi}^3}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\theta_j(x_1+2\tau_1\sqrt{\mu(t-\tau)}, x_2+2\tau_2\sqrt{\mu(t-\tau)}, x_3+2\tau_3\sqrt{\mu(t-\tau)};\tau)\times \\ \times d\tau_1d\tau_2d\tau_3d\tau]\times[H_{jx_i}^0 - \frac{1}{\sqrt{\pi}^3}\int_0^t\int_{R^3}\frac{\tau_i}{\sqrt{\mu(t-\tau)}}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\theta_j(x_1+2\tau_1\sqrt{\mu(t-\tau)}, x_2+2\tau_2\times \\ \times\sqrt{\mu(t-\tau)}, x_3+2\tau_3\sqrt{\mu(t-\tau)};\tau)d\tau_1d\tau_2d\tau_3d\tau]\} \equiv D_i[\theta_1,\theta_2,\theta_3], (s_i - x_i = 2\tau_i\sqrt{\mu(t-\tau)}; i=\overline{1,3}). \end{aligned} \tag{3.6}$$

Here for example, partial derivatives of functions υ_i are defined:

$$\begin{cases} \upsilon_{ix_j} = H_{ix_j}^0 - \frac{1}{8\sqrt{\pi^3}}\int_0^t\int_{R^3}\frac{-(x_j-s_j)}{2\mu(t-\tau)}\frac{1}{(\sqrt{\mu(t-\tau)})^3}\exp(-\frac{r^2}{4\mu(t-\tau)})\theta_i(s_1,s_2,s_3,\tau)ds_1ds_2ds_3d\tau = \\ = H_{ix_j}^0 - \frac{1}{\sqrt{\pi}^3}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\frac{\tau_j}{\sqrt{\mu(t-\tau)}}\theta_i(x_1+2\tau_1\sqrt{\mu(t-\tau)}, x_2+2\tau_2\sqrt{\mu(t-\tau)}, x_3 + \\ +2\tau_3\sqrt{\mu(t-\tau)};\tau)d\tau_1d\tau_2d\tau_3d\tau, (i=\overline{1,3};\ j=\overline{1,3}), \\ H_{ix_j}^0(x_1,x_2,x_3,t) \equiv \frac{1}{\sqrt{\pi}^3}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\frac{\tau_j}{\sqrt{\mu(t-\tau)}}\Phi_i(x_1+2\tau_1\sqrt{\mu(t-\tau)}, x_2+2\tau_2\times \\ \times\sqrt{\mu(t-\tau)}, x_3+2\tau_3\sqrt{\mu(t-\tau)};\tau)d\tau_1d\tau_2d\tau_3d\tau. \end{cases} \tag{3.7}$$

Here (3.6) – system of the nonlinear integrated equations of Volterra-Abel of the second sort concerning θ_i on a variable $t \in [0, T_0]$ and consists of three integral equations, and contains in itself of three unknown functions.

The theory of the specified system is well developed in section of mathematics [13]. Therefore there is no necessity to think out various algorithms for the decision of this system. And it is enough to show conditions which provide conditions of contraction mapping principle for the decision of this system to use a Picard's method.

If takes place

$$\begin{cases}
\forall (x_1, x_2, x_3, t) \in T; \Phi_i; H_i^0: \\
\sup\limits_T \left| D^k \Phi_i(x_1, x_2, x_3, t) \right| \le \gamma_0, \quad (i = \overline{1,3};\ k = \overline{0,2}), \\
\dfrac{1}{\sqrt{\pi^3}} \sup\limits_{[0,T_0]} \int\limits_0^t \int\limits_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2)) d\tau_1 d\tau_2 d\tau_3 d\tau \le T_0, \\
\dfrac{1}{\sqrt{\pi^3}} \sup\limits_T \int\limits_0^t \int\limits_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2)) \left| \Phi_i(x_1 + 2\tau_1 \sqrt{\mu(t-\tau)}, x_2 + 2\tau_2 \sqrt{\mu(t-\tau)}, x_3 + 2\tau_3 \times \right. \\
\left. \times \sqrt{\mu(t-\tau)}; \tau) \right| d\tau_1 d\tau_2 d\tau_3 d\tau \le \gamma_0 T_0, \\
\dfrac{1}{\sqrt{\mu}\sqrt{\pi^3}} \sup\limits_T \int\limits_0^t \int\limits_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2)) \dfrac{|\tau_i|}{\sqrt{t-\tau}} d\tau_1 d\tau_2 d\tau_3 d\tau \le (\sqrt{\mu})^{-1} \dfrac{1}{\sqrt{\pi^3}} \sup\limits_T \int\limits_0^t \dfrac{1}{\sqrt{t-\tau}} \times \\
\times \{ (\int\limits_{R^3} \tau_i^2 \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2)) d\tau_1 d\tau_2 d\tau_3)^{\frac{1}{2}} (\int\limits_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2)) d\tau_1 d\tau_2 d\tau_3)^{\frac{1}{2}} \} d\tau = \\
= (\sqrt{\mu})^{-1} \dfrac{1}{\sqrt{2}} \sup\limits_{[0,T_0]} \int\limits_0^t \dfrac{1}{\sqrt{t-\tau}} d\tau = (\sqrt{\mu})^{-1} \sqrt{2T_0}, \\
\left| H_{ix_j}^0 \right| \le \dfrac{1}{\sqrt{\mu}\sqrt{\pi^3}} \sup\limits_T \int\limits_0^t \int\limits_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2)) \dfrac{|\tau_j|}{\sqrt{t-\tau}} \left| \Phi_i(x_1 + 2\tau_1 \sqrt{\mu(t-\tau)}, x_2 + 2\tau_2 \sqrt{\mu(t-\tau)}, \right. \\
\left. x_3 + 2\tau_3 \sqrt{\mu(t-\tau)}; \tau) \right| d\tau_1 d\tau_2 d\tau_3 d\tau \le (\sqrt{\mu})^{-1} \gamma_0 \sqrt{2T_0},
\end{cases} \qquad (3.8)$$

and if operators: D_i compressing with a compression factor d_i,

$$\begin{cases}
D_i : d_i \le \dfrac{d}{3}, (d < 1), \ (i = \overline{1,3}), \\
\sum\limits_{i=1}^3 d_i \le d = 12(\sqrt{\mu})^{-1} \gamma_* < 1, \\
d_i = 4(\sqrt{\mu})^{-1} [2\sqrt{2}\gamma_0 \sqrt{T_0^3} + \sqrt{2} r_1 \sqrt{T_0^3}] \le 4(\sqrt{\mu})^{-1} \gamma_* < 1, \ (i = \overline{1,3}), \\
\gamma_* = 2\sqrt{2}\gamma_0 \sqrt{T_0^3} + \sqrt{2} r_1 \sqrt{T_0^3}, \quad (k_0 < \mu = \mu_0 < \infty, \ [k_0 > \max(1;\ 144\gamma_4^2)]), \\
S_{r_1}(\theta_i^0) = \{ \theta_i : \left| \theta_i - \theta_i^0 \right| \le r_1, \ \forall (x_1, x_2, x_3, t) \in T \},
\end{cases} \qquad (3.9)$$

and [13]:

$$\begin{cases} \left\| D_i[\theta_1^0,\theta_2^0,\theta_3^0]-\theta_i^0 \right\|_C \le r_1(1-d): \\ \left\| D_i[\theta_1,\theta_2,\theta_3]-\theta_i^0 \right\|_C \le \left\| D_i[\theta_1,\theta_2,\theta_3]-D_i[\theta_1^0,\theta_2^0,\theta_3^0] \right\|_C + \left\| D_i[\theta_1^0,\theta_2^0,\theta_3^0]-\theta_i^0 \right\|_C \le \\ \le d_i 3r_1 + r_1(1-d) \le dr_1 + r_1(1-d) = r_1, \\ D_i : S_{r_1}(\theta_i^0) \to S_{r_1}(\theta_i^0),\ (i=\overline{1,3}). \end{cases} \qquad (3.10)$$

Then on the basis of a contraction mapping principle the system (3.6) is solvable at $C^{2,0}(T)$. Hence the solution of this system we can find on the basis of Picard's method:

$$\theta_{i,n+1} \equiv D_i[\theta_{1,n},\theta_{2,n},\theta_{3,n}],(n=0,1,...;i=\overline{1,3}), \qquad (3.11)$$

where $\theta_{1,0},\theta_{2,0},\theta_{3,0}$ – initial estimates. Received the sequence of functions $\{\theta_{i,n}\}_0^\infty,(i=\overline{1,3})$ is converging and fundamental in $S_{r_1}(\theta_i^0)$:

$$\begin{cases} E_{n+1} = \sum_{i=1}^{3} \left\| \theta_{i,n+1}-\theta_{i,n} \right\|_C;\ E_n = \sum_{i=1}^{3} \left\| \theta_{i,n}-\theta_{i,n-1} \right\|_C,\ (i=\overline{1,3}): \\ \left\| \theta_{i,n+1}-\theta_{i,n} \right\|_C \le d_i \sum_{i=1}^{3} \left\| \theta_{i,n}-\theta_{i,n-1} \right\|_C = d_i E_n;\quad E_{n+1} \le dE_n \le ... \le d^n E_1 \xrightarrow[n\to\infty]{d<1} 0, \\ \left\| \theta_{i,n+k}-\theta_{i,n} \right\|_C \le \sum_{j=0}^{k-1} \left\| \theta_{i,n+j+1}-\theta_{i,n+j} \right\|_C \le \sum_{j=0}^{k-1} d_i \sum_{i=1}^{3} \left\| \theta_{i,n+j}-\theta_{i,n+j-1} \right\|_C = \sum_{j=0}^{k-1} d_i E_{n+j}, \\ E_{n+k} \le d\sum_{j=0}^{k-1} E_{n+j} \le ... \le d\sum_{j=0}^{k-1} d^{n+j-1}E_1 \le E_1 d^n \sum_{j=0}^{k-1} d^j \le E_1 d^n \frac{1}{1-d} \xrightarrow[n\to\infty]{d<1} 0, \end{cases}$$

and thus converging to a limit $\theta_i,(i=\overline{1,3})$:

$$\begin{cases} U_{n+1} = \sum_{i=1}^{3} \left\| \theta_{i,n+1}-\theta_i \right\|_C;\ U_0 = \sum_{i=1}^{3} \left\| \theta_i-\theta_{i,0} \right\|_C:\ U_{n+1} \le dU_n \le ... \le d^{n+1}U_0 \xrightarrow[n\to\infty]{d<1} 0, \\ \theta_{i,n+1} \xrightarrow[n\to\infty]{d<1} \theta_i \equiv \omega_i, \forall (x_1,x_2,x_3,t)\in T,\ (i=\overline{1,3}). \end{cases} \qquad (3.12)$$

Then according to results of the theorem 3, functions $\upsilon_i, i=\overline{1,3}$ are defined from system (3.5)

$$\begin{cases} \upsilon_i = \frac{1}{\sqrt{\pi}^3}\int_0^t \int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Phi_i(x_1+2\tau_1\sqrt{\mu(t-\tau)},x_2+2\tau_2\sqrt{\mu(t-\tau)},x_3+2\tau_3\times \\ \times\sqrt{\mu(t-\tau)};\tau)d\tau_1 d\tau_2 d\tau_3 d\tau - \frac{1}{\sqrt{\pi}^3}\int_0^t \int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\omega_i(x_1+2\tau_1\sqrt{\mu(t-\tau)},x_2+ \\ +2\tau_2\sqrt{\mu(t-\tau)},x_3+2\tau_3\sqrt{\mu(t-\tau)};\tau)d\tau_1 d\tau_2 d\tau_3 d\tau = \frac{1}{\sqrt{\pi}^3}\int_0^t \int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Phi_i^0(x_1+ \\ +2\tau_1\sqrt{\mu(t-\tau)},x_2+2\tau_2\sqrt{\mu(t-\tau)},x_3+2\tau_3\sqrt{\mu(t-\tau)};\tau)d\tau_1 d\tau_2 d\tau_3 d\tau \equiv H_i(x_1,x_2,x_3,t), \\ \Phi_i^0(x_1,x_2,x_3,t) \equiv \Phi_i - \omega_i, \\ s_i - x_i = 2\tau_i\sqrt{\mu(t-\tau)},(i=\overline{1,3}), \end{cases} \qquad (3.5)^*$$

here Φ_i, ω_i, H_i - known functions and

$$\begin{cases} \forall (x_1,x_2,x_3,t)\in T;\Phi_i^0: \quad \sup_T \left|D^k \Phi_i^0(x_1,x_2,x_3,t)\right| \le \overline{\beta_1}, \quad (i=\overline{1,3}; k=\overline{0,2}), \\ \sup_T \frac{1}{\sqrt{\pi^3}} \int_0^t \int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)\left|\Phi_i^0(l_1,l_2,l_3;\tau)\right| d\tau_1 d\tau_2 d\tau_3 d\tau \le \overline{\beta_2} = \overline{\beta_1} T_0, \\ \sup_T \frac{1}{\sqrt{\pi^3}} \int_0^t \int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)\frac{1}{\sqrt{t-s}} \sum_{j=1}^3 \left|\tau_j\right| \times \left|\Phi_{il_j^k}^{0(k)}(l_1,l_2,l_3;\tau)\right| d\tau_1 d\tau_2 d\tau_3 d\tau \le \overline{\beta_1}\sqrt{2T_0} = \overline{\beta_3}, \\ l_j = x_j + 2\tau_j \sqrt{\mu(t-s)}, (j=\overline{1,3};\ k=\overline{0,2}), \quad \beta_0 = \beta(1+\sqrt{\mu}), \quad \beta = \max_{1\le i\le 3} \overline{\beta_i}, \\ \left|\upsilon_i\right| = \frac{1}{\sqrt{\pi^3}} \sup_T \int_0^t \int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\left|\Phi_i^0(x_1+2\tau_1\sqrt{\mu(t-\tau)}, x_2+2\tau_2\sqrt{\mu(t-\tau)}, \right. \\ \left. x_3+2\tau_3\sqrt{\mu(t-\tau)};\tau)\right| d\tau_1 d\tau_2 d\tau_3 d\tau \le \overline{\beta_1} T_0 \le \beta, \ \forall (x_1,x_2,x_3,t)\in T, (i=\overline{1,3}). \end{cases} \tag{3.13}$$

Hence

$$\left\|\upsilon_i\right\|_{C^{3,0}(T)} \le 20\beta, (\left\|\upsilon_i\right\|_{C(T)} \le \beta; \quad i=\overline{1,3}).$$

Then considering norm of space $\tilde{C}^{3,1}_{n=3}(T)$ we will receive

$$\begin{cases} \left\|v\right\|_{\tilde{C}^{3,1}_{n=3}(T)} = \sum_{i=1}^3 \{\left\|\upsilon_i\right\|_{C^{3,0}(T)} + \left\|\upsilon_{it}\right\|_{C(T)}\} \le 3[N_1+\beta_0] = M^*, \\ \left\|\upsilon_i\right\|_{C^{3,0}(T)} = \sum_{0\le|k|\le 3} \left\|D^k \upsilon_i\right\|_{C(T)} \le N_1 = 20\beta, (i=\overline{1,3}), \\ \left\|\upsilon_{it}\right\|_{C(T)} \le \beta_0 = \beta(1+\sqrt{\mu}), (i=\overline{1,3}). \end{cases} \tag{3.14}$$

Thus (3.5)* satisfies the equation (3.4):

$$\upsilon_{it} = \Phi_i^0 + \mu\Delta\upsilon_i, (i=\overline{1,3}), \tag{3.4*}$$

where

$$\begin{cases} \Phi_i^0 \equiv \Phi_i - \omega_i; \quad \sum_{i=1}^3 \upsilon_{ix_i} = \sum_{i=1}^3 H_{ix_i} = 0: \\ \sum_{i=1}^3 \Phi^0_{ix_i} = 0, \ \forall (x_1,x_2,x_3,t)\in T; \quad \operatorname{div}\tilde{\omega} = 0, \ (\tilde{\omega} = (\omega_1,\omega_2,\omega_3); \ \theta_i \equiv \omega_i), \\ \Phi_i \equiv f_i - J_{0x_i}, (i=\overline{1,3}), \quad \sum_{i=1}^3 \Phi_{ix_i} \equiv F_0 + \Delta J_0 = 0. \end{cases}$$

Really, having calculated partial derivative of system (3.5)*:

$$\begin{cases} k_0 < \mu = \mu_0 = \text{const} < \infty, \ (k_0 > \max(1;\ 144\gamma_*^2)); \forall (x_1,x_2,x_3,t)\in T: \\ \upsilon_i = \frac{1}{8\sqrt{\pi^3}} \int_0^t \int_{R^3} \exp(-\frac{r^2}{4\mu(t-\tau)})\Phi_i^0(s_1,s_2,s_3,\tau)\frac{ds_1 ds_2 ds_3 d\tau}{(\sqrt{\mu(t-\tau)})^3} = \frac{1}{\sqrt{\pi}^3} \int_0^t \int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\times \end{cases}$$

$$\begin{cases} \times\Phi_i^0(x_1+2\tau_1\sqrt{\mu(t-\tau)},x_2+2\tau_2\sqrt{\mu(t-\tau)},x_3+2\tau_3\sqrt{\mu(t-\tau)};\tau)d\tau_1 d\tau_2 d\tau_3 d\tau,(i=\overline{1,3}), \\ \upsilon_{it}=\Phi_i^0(x_1,x_2,x_3,t)+\frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\sum_{j=1}^3\sqrt{\mu}\frac{\tau_j}{\sqrt{t-\tau}}\Phi_{il_j}^0(x_1+2\tau_1\sqrt{\mu(t-\tau)},x_2+ \\ +2\tau_2\sqrt{\mu(t-\tau)},x_3+2\tau_3\sqrt{\mu(t-\tau)};\tau)d\tau_1 d\tau_2 d\tau_3 d\tau,\ (l_j=x_j+2\tau_j\sqrt{\mu(t-\tau)};\ i=\overline{1,3};\ j=\overline{1,3}), \\ \upsilon_{ix_j}=\frac{1}{8\sqrt{\pi^3}}\int_0^t\int_{R^3}\frac{-(x_j-s_j)}{2\mu(t-\tau)}\frac{1}{(\sqrt{\mu(t-\tau)})^3}\exp(-\frac{r^2}{4\mu(t-\tau)})\Phi_i^0(s_1,s_2,s_3,\tau)ds_1 ds_2 ds_3 d\tau= \\ =\frac{1}{\sqrt{\pi}^3}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\frac{\tau_j}{\sqrt{\mu(t-\tau)}}\Phi_i^0(x_1+2\tau_1\sqrt{\mu(t-\tau)},x_2+2\tau_2\sqrt{\mu(t-\tau)},x_3+ \\ +2\tau_3\sqrt{\mu(t-\tau)};\tau)d\tau_1 d\tau_2 d\tau_3 d\tau, \\ \upsilon_{ix_j^2}=\frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\frac{\tau_j}{\sqrt{\mu(t-\tau)}}\Phi_{il_j}^0(x_1+2\tau_1\sqrt{\mu(t-\tau)},x_2+2\tau_2\sqrt{\mu(t-\tau)},x_3+ \\ +2\tau_3\sqrt{\mu(t-\tau)};\tau)d\tau_1 d\tau_2 d\tau_3 d\tau, \\ \mu\Delta\upsilon_i=\frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\sum_{j=1}^3\sqrt{\mu}\frac{\tau_j}{\sqrt{t-\tau}}\Phi_{il_j}^0(x_1+2\tau_1\sqrt{\mu(t-\tau)},x_2+2\tau_2\sqrt{\mu(t-\tau)}, \\ x_3+2\tau_3\sqrt{\mu(t-\tau)};\tau)d\tau_1 d\tau_2 d\tau_3 d\tau, \end{cases} \tag{3.15}$$

and substituting (3.15) in (3.4)*, we have

$$\begin{cases} \upsilon_i\big|_{t=0}=0,\forall(x_1,x_2,x_3)\in R^3;\ \ 1<\mu=\mu_0<\infty;\ \ \forall(x_1,x_2,x_3,t)\in T: \\ 0=\upsilon_{it}-\Phi_i^0-\mu\Delta\upsilon_i\equiv\Phi_i^0+\frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\sum_{j=1}^3\frac{\sqrt{\mu}\tau_j}{\sqrt{t-\tau}}\Phi_{il_j}^0(x_1+2\tau_1\sqrt{\mu(t-\tau)}, \\ x_2+2\tau_2\sqrt{\mu(t-\tau)},x_3+2\tau_3\sqrt{\mu(t-\tau)};\tau)d\tau_1 d\tau_2 d\tau_3 d\tau-\Phi_i^0-\frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\times \\ \times\sum_{j=1}^3\frac{\sqrt{\mu}\tau_j}{\sqrt{t-\tau}}\Phi_{il_j}^0(x_1+2\tau_1\sqrt{\mu(t-\tau)},x_2+2\tau_2\sqrt{\mu(t-\tau)},x_3+2\tau_3\sqrt{\mu(t-\tau)};\tau)d\tau_1 d\tau_2 d\tau_3 d\tau=0. \end{cases} \tag{3.16}$$

That it was required to show.

From the received results, on the basis of (3.3) follows

$$\frac{1}{\rho}P=-\frac{1}{2}Q+\frac{1}{4\pi}\int_{R^3}\frac{F_0(s_1,s_2,s_3,t)ds_1 ds_2 ds_3}{r}. \tag{3.17}$$

Then according to results of the theorem 3, functions $\upsilon_i, i=\overline{1,3}$ are defined from system (3.5)* and satisfies the equation (1.2). For a problem Navier-Stokes (1.1)-(1.3)*, (A_2), are proved: existence of the smooth single solution in area $\tilde{C}_{n=3}^{3,1}(T)$, and we will notice that the received decision (3.5)* continuously depends on the initial data $f_i,(i=\overline{1,3})$. The theorem is proved. ■

4. Fluid with Very Small Viscosity with a Condition (A_3)

In the theory of the differential equations in partial derivatives there are various mathematical transformations which simplify investigated problems and does possible to find the decision in certain spaces [6, 12, 13 and 15]. Here in a case $0 < \mu < 1$ (Reynolds number [12]: **Re** ≥ 2300) we will show that at certain mathematical transformations of the equation Navier-Stokes is led to a linear kind. At that the new system has an analytical solution, which is based on the Picard's method.

So, inverse Fourier transform plays great part while deciding boundary problem for the solution of some integral equations, integration; Laplace transformation – while solution of simple differential equation of multiple of *N* with the constant rate and their systems, some differential equation in the partial derivative, Volterra equation of the second and first type with the difference kernel, and integral equation with the logarithmic kernel and etc. But at the decision of equations Navier-Stokes in the general form these transformations yet have not brought desirable results, if we do not consider special cases.

Therefore for the last decades in the mathematics the methods, connected with using of integral of the transform, became widely spread. Different formulas of integral transform arise while a concrete problem solving, but in the sequel they can be applied to the solution of other problems when researching the differential and integral equation, integration.

From the received results follows that system Navier-Stokes (1.1) in the conditions of (1.2), (1.3), (A_3) can have the analytical smooth single solution. At least, such decision answers a mathematical question, and possibility to construct the solution on a problem Navier-Stokes (1.1) - (1.3) for an incompressible liquid with viscosity with a condition (A_3).

4.1. Fluid with Viscosity $0 < \mu < 1$, when $\operatorname{div} f \neq 0$

Let υ_{i0} initial components of a vector of speed v at the moment of time t=0 it is set in a kind (1.3):

$$\upsilon_i\big|_{t=0} = \upsilon_{i0}(x_1, x_2, x_3) \equiv \lambda_i \vartheta_0(x_1, x_2, x_3), (i = \overline{1,3}), \tag{4.1}$$

where $0<\lambda_i$ – the known constants. Then speed components v are defined by a rule

$$\begin{cases} \upsilon_i = \lambda_i V(x_1, x_2, x_3, t), (i = \overline{1,3}), \\ V\big|_{t=0} = \vartheta_0(x_1, x_2, x_3), \ \forall (x_1, x_2, x_3) \in R^3, \\ \operatorname{div} f \neq 0; \ \operatorname{div} v = 0: \\ \sum\limits_{j=1}^{3} \lambda_j V_{x_j} = 0; \ \sum\limits_{j=1}^{3} \upsilon_j \upsilon_{ix_j} = \lambda_i V \sum\limits_{j=1}^{3} \lambda_j V_{x_j} = 0. \end{cases} \tag{4.2}$$

Hence, the system (1.1) will be transformed to a kind

$$\lambda_i V_t = f_i - \frac{1}{\rho} P_{x_i} + \mu \lambda_i \Delta V, (i = \overline{1,3}), \tag{4.3}$$

where V new unknown function which defines the decision on problem Navier-Stokes. Here substitution

(4.2) it is equivalent will transform system (1.1) to the nonhomogeneous linear equation of a kind (4.3). At that (4.1)-(4.3) are investigated in work [8] in $G^2_\lambda(D_0)$ or $W^2_\lambda(D_0)$.

Here problems (4.1)-(4.3) it is investigated in $G^1_{n=3}(D_0)$. For this purpose, at first we will define pressure P. Really, considering APS from system (4.3) we will receive:

$$\begin{cases} \sum_{i=1}^{3} \frac{\partial}{\partial x_i}(4.3): \quad \frac{1}{\rho}\Delta P = -F_0, (F_0 \equiv -\sum_{i=1}^{3} f_{ix_i}(x_1,x_2,x_3,t)), \\ \frac{1}{\rho}P = \frac{1}{4\pi}\int_{R^3} F_0(s_1,s_2,s_3,t)\frac{ds_1 ds_2 ds_3}{r}, \\ \frac{1}{\rho}P_{x_i} = \frac{1}{4\pi}\int_{R^3} \frac{\tau_i F_0(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t)d\tau_1 d\tau_2 d\tau_3}{\sqrt{(\tau_1^2+\tau_2^2+\tau_3^2)^3}}, (s_i - x_i = \tau_i; i=\overline{1,3}). \end{cases} \tag{4.4}$$

Therefore

$$\begin{cases} V_t = \Phi_0(x_1,x_2,x_3,t) + \mu\Delta V, \ \forall(x_1,x_2,x_3,t)\in T, \\ \sum_{i=1}^{3}\Phi_{0x_i} = 0, \ \forall(x_1,x_2,x_3,t)\in T, \\ (\lambda_1)^{-1}(f_1 - \rho^{-1}P_{x_1}) = (\lambda_2)^{-1}(f_2-\rho^{-1}P_{x_2}) = (\lambda_3)^{-1}(f_3-\rho^{-1}P_{x_3}) \equiv \Phi_0, \end{cases} \tag{4.5}$$

i.e. is the system (4.3) is transformed to the linear equations of heat conductivity with a condition of Cauchy in a kind (4.5), and in a class of functions with smooth enough initial data is correctly put [13, 14]. Accordingly there is an the conditional-smooth and single solution of a problem Navier-Stokes in $G^1(D_0)$.

Really from system (4.13), follows:

$$\begin{aligned} V &= \frac{1}{8(\sqrt{\pi\mu t})^3}\int_{R^3}\exp(-\frac{r^2}{4\mu t})\vartheta_0(s_1,s_2,s_3)ds_1ds_2ds_3 + \frac{1}{8\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-\frac{r^2}{4\mu(t-s)})\frac{1}{\sqrt{(\mu(t-s))^3}}\times \\ &\times\Phi_0(s_1,s_2,s_3,s)ds_1ds_2ds_3ds = \frac{1}{\sqrt{\pi^3}}\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\vartheta_0(x_1+2\tau_1\sqrt{\mu t},x_2+2\tau_2\sqrt{\mu t},x_3+2\tau_3\times \\ &\times\sqrt{\mu t})d\tau_1d\tau_2d\tau_3 + \frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Phi_0(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+ \\ &+2\tau_3\sqrt{\mu(t-s)};s)d\tau_1d\tau_2d\tau_3ds \equiv H_0(x_1,x_2,x_3,t),(s_i-x_i=2\tau_i\sqrt{\mu t};s_i-x_i=2\tau_i\sqrt{\mu(t-s)};i=\overline{1,3}), \end{aligned} \tag{4.6}$$

H_0 – is known function. The found decision (4.6) satisfies system (4.5).

Really, considering partial derivative systems (4.6):

$$\begin{cases} (0,1)\ni\mu; \ \forall(x_1,x_1,x_1,t)\in T: \\ V_{x_j} = \frac{1}{\sqrt{\pi^3}}\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\vartheta_{0h_j}(x_1+2\tau_1\sqrt{\mu t},x_2+2\tau_2\sqrt{\mu t},x_3+2\tau_3\sqrt{\mu t})d\tau_1d\tau_2d\tau_3 + \end{cases}$$

$$
\begin{cases}
+\frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Phi_{0l_j}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\times \\
\times\sqrt{\mu(t-s)};s)d\tau_1 d\tau_2 d\tau_3 ds, \\
V_{x_j^2}=\frac{1}{\sqrt{\pi^3}}\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\vartheta_{0h_j^2}(x_1+2\tau_1\sqrt{\mu t},x_2+2\tau_2\sqrt{\mu t},x_3+2\tau_3\sqrt{\mu t})d\tau_1 d\tau_2 d\tau_3+ \\
+\frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Phi_{0l_j^2}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\times \\
\times\sqrt{\mu(t-s)};s)d\tau_1 d\tau_2 d\tau_3 ds, \\
\forall(x_1,x_2,x_3)\in R^3;\quad t\in(0,T_0]: \\
V_t=\frac{1}{\sqrt{\pi^3}}\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\times(\sum_{j=1}^{3}\sqrt{\mu}\frac{\tau_j}{\sqrt{t}}\vartheta_{0h_j}(x_1+2\tau_1\sqrt{\mu t},x_2+2\tau_2\sqrt{\mu t},x_3+ \\
+2\tau_3\sqrt{\mu t})d\tau_1 d\tau_2 d\tau_3+\Phi_0+\frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\sum_{j=1}^{3}\sqrt{\mu}\frac{\tau_j}{\sqrt{t-s}}\Phi_{0l_j}(x_1+2\tau_1\sqrt{\mu(t-s)}, \\
x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)d\tau_1 d\tau_2 d\tau_3 ds, \\
h_j=x_j+2\tau_j\sqrt{\mu t};\ l_j=x_j+2\tau_j\sqrt{\mu(t-s)},\ \ (j=\overline{1,3}),
\end{cases}
\tag{4.7}
$$

and substituting (4.7) in (4.5), we have

$$
\begin{cases}
V\big|_{t=0}=\vartheta_0(x_1,x_2,x_3),\ \forall(x_1,x_2,x_3)\in R^3;\quad (0,1)\ni\mu;\ \forall(x_1,x_2,x_3)\in R^3;\quad t\in(0,T_0]: \\
0=V_t-\Phi_0-\mu\Delta V_0\equiv\frac{1}{\sqrt{\pi^3}}\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\times(\sum_{j=1}^{3}\sqrt{\mu}\frac{\tau_j}{\sqrt{t}}\vartheta_{0h_j}(x_1+2\tau_1\sqrt{\mu t},x_2+2\tau_2\sqrt{\mu t},x_3+ \\
+2\tau_3\sqrt{\mu t}))d\tau_1 d\tau_2 d\tau_3+\Phi_0+\frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\times(\sum_{j=1}^{3}\sqrt{\mu}\frac{\tau_j}{\sqrt{t-s}}\Phi_{0l_j}(x_1+2\tau_1\sqrt{\mu(t-s)}, \\
x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s))d\tau_1 d\tau_2 d\tau_3 ds-\Phi_0-\mu\{\frac{1}{\sqrt{\pi^3}}\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\times \\
\times\Delta\vartheta_0(x_1+2\tau_1\sqrt{\mu t},x_2+2\tau_2\sqrt{\mu t},x_3+2\tau_3\sqrt{\mu t})d\tau_1 d\tau_2 d\tau_3+\frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Delta\Phi_0(x_1+ \\
+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)d\tau_1 d\tau_2 d\tau_3 ds\}=\frac{1}{\sqrt{\pi^3}}\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\times \\
\times(\sum_{j=1}^{3}\sqrt{\mu}\frac{\tau_j}{\sqrt{t}}\vartheta_{0h_j}(x_1+2\tau_1\sqrt{\mu t},x_2+2\tau_2\sqrt{\mu t},x_3+2\tau_3\sqrt{\mu t}))d\tau_1 d\tau_2 d\tau_3+\frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\times \\
\times(\sum_{j=1}^{3}\sqrt{\mu}\frac{\tau_j}{\sqrt{t-s}}\Phi_{0l_j}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s))d\tau_1 d\tau_2 d\tau_3 ds- \\
-\frac{1}{2}\sqrt{\mu}\{\frac{1}{\sqrt{\pi^3}}\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\frac{1}{\sqrt{t}}\vartheta_{0h_1^2}(x_1+2\tau_1\sqrt{\mu t},x_2+2\tau_2\sqrt{\mu t},x_3+2\tau_3\sqrt{\mu t})d(x_1+ \\
+2\tau_1\sqrt{\mu t})d\tau_2 d\tau_3+\frac{1}{\sqrt{\pi^3}}\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\frac{1}{\sqrt{t}}\vartheta_{0h_2^2}(x_1+2\tau_1\sqrt{\mu t},x_2+2\tau_2\sqrt{\mu t},x_3+2\tau_3\sqrt{\mu t})\times
\end{cases}
$$

$$
\begin{cases}
\times d\tau_1 d(x_2+2\tau_2\sqrt{\mu t})d\tau_2 d\tau_3+\frac{1}{\sqrt{\pi^3}}\int\limits_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\frac{1}{\sqrt{t}}\vartheta_{0h_3^2}(x_1+2\tau_1\sqrt{\mu t},x_2+2\tau_2\sqrt{\mu t},x_3+\\
+2\tau_3\sqrt{\mu t})d\tau_1 d\tau_2 d(x_3+2\tau_3\sqrt{\mu t})+\frac{1}{\sqrt{\pi^3}}\int\limits_0^t\frac{1}{\sqrt{t-s}}[\int\limits_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Phi_{0l_1^2}(x_1+2\tau_1\sqrt{\mu(t-s)},\\
x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)d(x_1+2\tau_1\sqrt{\mu(t-s)})d\tau_2 d\tau_3+\int\limits_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\times\\
\times\Phi_{0l_2^2}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)d\tau_1 d(x_2+2\tau_2\sqrt{\mu(t-s)})d\tau_3+\\
+\int\limits_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Phi_{0l_3^2}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)\times\\
\times d\tau_1 d\tau_2 d(x_3+2\tau_3\sqrt{\mu(t-s)})]ds\}=\frac{1}{\sqrt{\pi^3}}\int\limits_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))(\sum_{j=1}^{3}\sqrt{\mu}\frac{\tau_j}{\sqrt{t}}\vartheta_{0h_j}(x_1+2\tau_1\sqrt{\mu t},\\
x_2+2\tau_2\sqrt{\mu t},x_3+2\tau_3\sqrt{\mu t}))d\tau_1 d\tau_2 d\tau_3+\frac{1}{\sqrt{\pi^3}}\int\limits_0^t\int\limits_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\times(\sum_{j=1}^{3}\sqrt{\mu}\frac{\tau_j}{\sqrt{t-s}}\Phi_{0l_j}(x_1+\\
+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s))d\tau_1 d\tau_2 d\tau_3 ds-\frac{1}{\sqrt{\pi^3}}\int\limits_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\times\\
\times(\sum_{j=1}^{3}\sqrt{\mu}\frac{\tau_j}{\sqrt{t}}\vartheta_{0h_j}(x_1+2\tau_1\sqrt{\mu t},x_2+2\tau_2\sqrt{\mu t},x_3+2\tau_3\sqrt{\mu t}))d\tau_1 d\tau_2 d\tau_3-\frac{1}{\sqrt{\pi^3}}\int\limits_0^t\int\limits_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\times\\
\times(\sum_{j=1}^{3}\sqrt{\mu}\frac{\tau_j}{\sqrt{t-s}}\Phi_{0l_j}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s))d\tau_1 d\tau_2 d\tau_3 ds\}=0,
\end{cases}
$$

i.e. the system (4.6) satisfies a problem (4.5). That it was required to prove.

The limiting case in $G^1(D_0)$, when the decision of (4.5) is representing in the form of (4.6), when

$$
\begin{cases}
\forall(x_1,x_2,x_3,t)\in T;V_0;\Phi_0: \quad \sup\limits_{R^3}\left|D^k V_0\right|\le\beta_1,\ (k=\overline{0,3});\quad \sup\limits_{R^3}\left|D^k\Phi_0(x_1,x_2,x_3,t)\right|\le\gamma_1,\\
\sup\limits_{T}\frac{1}{\sqrt{\pi^3}}\int\limits_0^t\int\limits_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2)\left|D^k\Phi_0(l_1,l_2,l_3;s)\right|d\tau_1 d\tau_2 d\tau_3 ds\le\gamma_1 T_0=\beta_2,\\
\sup\limits_{T}\frac{1}{\sqrt{\pi^3}}\int\limits_0^t\int\limits_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2)\frac{1}{\sqrt{t-s}}\sum_{j=1}^{3}\left|\tau_j\right|\times\left|\Phi_{0l_j}(l_1,l_2,l_3;s)\right|d\tau_1 d\tau_2 d\tau_3 ds\le 3\gamma_1\sqrt{2T_0}=\beta_3,\\
l_i=x_i+2\tau_i\sqrt{\mu(t-s)},(i=\overline{1,3}),\quad \sup\limits_{R^3}\int\limits_0^{T_0}\left|\Phi_0(x_1,x_2,x_3,s)\right|ds\le\gamma_1 T_0=\beta_2,\\
\sup\limits_{R^3}\frac{1}{\sqrt{\pi^3}}\int\limits_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))(\sum_{i=1}^{3}\left|\tau_j\right|\times\left|V_{0\bar{l}_j}(\bar{l}_1,\bar{l}_2,\bar{l}_3)\right|)d\tau_1 d\tau_2 d\tau_3\le\beta_1\frac{1}{\sqrt{\pi^3}}\times\\
\times\{\sum_{i=1}^{3}(\int\limits_{R^3}\tau_i^2\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))d\tau_1 d\tau_2 d\tau_3)^{\frac{1}{2}}(\int\limits_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))d\tau_1 d\tau_2 d\tau_3)^{\frac{1}{2}}\}=\\
=3\beta_1\frac{1}{\sqrt{2}},\quad (\bar{l}_i=x_i+2\tau_i\sqrt{\mu t};\ i=\overline{1,3}),\ \beta=\max\limits_{1\le i\le 3}\beta_i;\quad \bar{\beta}_0=\beta(3\sqrt{2\mu T_0}+1+T_0\sqrt{\mu}).
\end{cases}
\tag{4.8}
$$

Really, estimating (4.6) in $G^1(D_0)$, we have

$$\begin{cases} \|V\|_{G^1(D_0)} = \|V\|_{C^{3,\beta}(T)} + \|V_t\|_{L^1} \le N_3 + \overline{\beta_0}, \\ \|V\|_{C^{3,\beta}(T)} = \sum_{0\le|k|\le 3} \|D^k V\|_{C(T)} \le N_3 = 40\beta, \\ \|V\|_{C(T)} \le 2\beta, \\ \|V_t\|_{L^1} = \sup_{R^3} \int_0^{T_0} |V_t(x_1,x_2,x_3,t)|dt \le \beta(3\sqrt{2\mu T_0} + 1 + T_0\sqrt{\mu}) = \overline{\beta_0}. \end{cases}$$

The singleness of the solution the system (4.6) in $G^1(D_0)$ is obvious on the basis of proof by contradiction [13]. Results (4.6) with a condition (4.2), (4.8) are received where smoothness of functions is required only on x_i as the derivative of 1st order is in time has $t>0$.

Hence, on a basis transformation (4.2) we will receive decisions of system (1.1), which satisfies a condition (1.2), i.e.

$$\begin{cases} \upsilon_i = \lambda_i H_0(x_1,x_2,x_3,t), \quad (i = \overline{1,3}), \\ \sum_{i=1}^{3} \upsilon_{ix_i} = \sum_{i=1}^{3} \lambda_i H_{0x_i} = 0, \\ \sum_{i=1}^{3} \lambda_i H_{0x_i} \equiv \frac{1}{\sqrt{\pi^3}} \int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)) \sum_{i=1}^{3} \lambda_i V_{0h_i}(x_1+2\tau_1\sqrt{\mu t}, x_2+2\tau_2\sqrt{\mu t}, x_3+2\tau_3\sqrt{\mu t}) \times \\ \times d\tau_1 d\tau_2 d\tau_3 + \frac{1}{\sqrt{\pi^3}} \int_0^t \int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)) \sum_{i=1}^{3} \lambda_i \Phi_{0l_i}(x_1+2\tau_1\sqrt{\mu(t-s)}, x_2+2\tau_2\sqrt{\mu(t-s)}, x_3 + \\ +2\tau_3\sqrt{\mu(t-s)}; s)d\tau_1 d\tau_2 d\tau_3 ds = 0, \quad (h_i = x_i + 2\tau_i\sqrt{\mu t};\ l_i = x_i + 2\tau_i\sqrt{\mu(t-s)}; i = \overline{1,3}). \end{cases} \tag{4.9}$$

In the conclusion estimating (4.9) it is had

$$\begin{cases} v = (\upsilon_1,\upsilon_2,\upsilon_3); \quad \upsilon_i = \lambda_i V, (i = \overline{1,3}): \\ \|v\|_{G^1_{n=3}(D_0)} = \sum_{i=1}^{3} [\|\lambda_i V\|_{C^{3,\beta}(T)} + \|\lambda_i V_t\|_{L^1}] \le d_0[N_3 + \overline{\beta_0}] = d_0 M_0, (d_0 = \sum_{i=1}^{3} \lambda_i; M_0 = N_3 + \overline{\beta_0}), \\ \|V\|_{G^1(D_0)} = \|V\|_{C^{3,\beta}(T)} + \|V_t\|_{L^1} \le N_3 + \overline{\beta_0}. \end{cases} \tag{4.10}$$

Theorem 4. In the conditions of (1.2), (4.1), (4.8) and (4.10) the problem (1.1), (1.2), (4.1) has a single solution in $G^1_{n=3}(D_0)$, which is defined by a rule (4.9).

4.2. Fluid with Small Viscosity, when $\Delta\vartheta_0 = 0;\ \operatorname{div} f \ne 0$

I. The overall objective of this point: to change a method (4.2) so that the received analytical solution of a problem Navier-Stokes with viscosity, belonged in $\tilde{C}^{3,1}_{n=3}(T)$.

If takes place

$$\begin{cases} \upsilon_i\big|_{t=0} = \upsilon_{i0}(x_1,x_2,x_3) \equiv \lambda_i \vartheta_0(x_1,x_2,x_3), i=\overline{1,3}, \\ \sum\limits_{j=1}^{3} \lambda_j \vartheta_{0x_j} = 0; \ \ \Delta\vartheta_0 = 0; \ \vartheta_0 \in C^3(R^3), \\ \operatorname{div} f \neq 0; \ \sup\limits_T \left|D^k f_i\right| \leq N_0 = \text{const}, \ (i=\overline{1,3}; k=\overline{0,4}), \end{cases} \tag{4.11}$$

that we will use transformation of a kind

$$\begin{cases} \upsilon_i = \lambda_i[\vartheta_0(x_1,x_2,x_3) + Z(x_1,x_2,x_3,t)], \forall(x_1,x_2,x_3,t)\in T, (i=\overline{1,3}), \\ Z\big|_{t=0} = 0, \ \forall(x_1,x_2,x_3)\in R^3, \\ \operatorname{div} v = 0: \ \sum\limits_{j=1}^{3}\lambda_j Z_{x_j} = 0; \ \sum\limits_{j=1}^{3}\lambda_j \vartheta_{0x_j} = 0, \\ \sum\limits_{j=1}^{3} \upsilon_j \upsilon_{ix_j} = \lambda_i\vartheta_0 \sum\limits_{j=1}^{3}\lambda_j\vartheta_{0x_j} + \lambda_i\vartheta_0\sum\limits_{j=1}^{3}\lambda_j Z_{x_j} + \lambda_i Z\sum\limits_{j=1}^{3}\lambda_j\vartheta_{0x_j} + \lambda_i Z\sum\limits_{j=1}^{3}\lambda_j Z_{x_j} = 0, \end{cases} \tag{4.12}$$

where $0 < \lambda_i -$ the known constants. Hence, the system (1.1) will be transformed to a kind

$$\lambda_i Z_t = f_i - \frac{1}{\rho}P_{x_i} + \mu\lambda_i \Delta Z, i=\overline{1,3}. \tag{4.13}$$

From system (4.13), considering conditions (4.11), (4.12), and having entered [8] APS we have the equation

$$\begin{cases} \sum\limits_{i=1}^{3}\frac{\partial}{\partial x_i}(4.13): \quad \frac{1}{\rho}\Delta P = -F_0, (F_0 \equiv -\sum\limits_{i=1}^{3} f_{ix_i}(x_1,x_2,x_3,t)), \\ \frac{1}{\rho}P = \frac{1}{4\pi}\int\limits_{R^3} F_0(s_1,s_2,s_3,t)\frac{ds_1 ds_2 ds_3}{r}, (r=\sqrt{(x_1-s_1)^2+(x_2-s_2)^2+(x_3-s_3)^2}), \\ \frac{1}{\rho}P_{x_i} = \frac{1}{4\pi}\int\limits_{R^3}\frac{\tau_i F_0(x_1+\tau_1, x_2+\tau_2, x_3+\tau_3; t)d\tau_1 d\tau_2 d\tau_3}{\sqrt{(\tau_1^2+\tau_2^2+\tau_3^2)^3}}, \ (s_i - x_i = \tau_i; \ i=\overline{1,3}). \end{cases} \tag{4.14}$$

Hence the system (4.13) will be transformed to a kind

$$\begin{cases} Z_t = \Phi_0 + \mu\Delta Z, \ \forall(x_1,x_2,x_3,t)\in T, \\ Z\big|_{t=o} = 0, \\ (\lambda_1)^{-1}(f_1 - \rho^{-1}P_{x_1}) = (\lambda_2)^{-1}(f_2-\rho^{-1}P_{x_2}) = (\lambda_3)^{-1}(f_3-\rho^{-1}P_{x_3}) \equiv \Phi_0(x_1,x_2,x_3,t). \end{cases} \tag{4.13*}$$

Then the decision of a problem (4.13)* is presented in a kind

$$\begin{aligned} Z &= \frac{1}{8\sqrt{\pi^3}}\int\limits_0^t\int\limits_{R^3}\exp\left(-\frac{r^2}{4\mu(t-s)}\right)\frac{1}{(\sqrt{\mu(t-s)})^3}\Phi_0(s_1,s_2,s_3,s)ds_1ds_2ds_3ds = \\ &= \frac{1}{\sqrt{\pi^3}}\int\limits_0^t\int\limits_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Phi_0(x_1+2\tau_1\sqrt{\mu(t-s)}, x_2+2\tau_2\sqrt{\mu(t-s)}, x_3+2\tau_3\times \\ &\times\sqrt{\mu(t-s)}; s)d\tau_1 d\tau_2 d\tau_3 ds \equiv H(x_1,x_2,x_3,t), \forall(x_1,x_2,x_3,t)\in T, (s_i - x_i = 2\tau_i\sqrt{\mu(t-s)}; i=\overline{1,3}), \end{aligned} \tag{4.15}$$

here H – known function. The solution (4.15) satisfies system (4.13)*.

Really, having calculated partial derivative of system (4.15):

$$\begin{cases} Z_t = \Phi_0(x_1,x_2,x_3,t) + \dfrac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\sum_{j=1}^{3}\sqrt{\mu}\,\dfrac{\tau_j}{\sqrt{t-s}}\Phi_{0l_j}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+ \\ +2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)d\tau_1 d\tau_2 d\tau_3 ds, \\ Z_{x_j} = \dfrac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Phi_{0l_j}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\times \\ \times\sqrt{\mu(t-s)};s)d\tau_1 d\tau_2 d\tau_3 ds, \\ Z_{x_j^2} = \dfrac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Phi_{0l_j^2}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\times \\ \times\sqrt{\mu(t-s)};s)d\tau_1 d\tau_2 d\tau_3 ds,\ (l_j = x_j + 2\tau_j\sqrt{\mu(t-s)};\ j=\overline{1,3}), \end{cases} \tag{4.16}$$

and substituting (4.16) in (4.13)*, we have

$$\begin{cases} Z\big|_{t=0} = 0,\ \ \forall(x_1,x_2,x_3)\in R^3, \\ (0,1)\ni\mu;\quad l_j = x_j+2\tau_j\sqrt{\mu(t-s)};\quad \forall(x_1,x_2,x_3,t)\in T: \\ 0 = Z_t - \Phi_0 - \mu\Delta Z \equiv \Phi_0 + \dfrac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\sum_{j=1}^{3}\sqrt{\mu}\,\dfrac{\tau_j}{\sqrt{t-s}}\Phi_{0l_j}(x_1+2\tau_1\sqrt{\mu(t-s)}, \\ x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)d\tau_1 d\tau_2 d\tau_3 ds - \Phi_0 - \mu\{\dfrac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\times \\ \times\sum_{j=1}^{3}\Phi_{0l_j^2}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)d\tau_1 d\tau_2 d\tau_3 ds\} = \\ = \dfrac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\sum_{j=1}^{3}\sqrt{\mu}\,\dfrac{\tau_j}{\sqrt{t-s}}\Phi_{0l_j}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+ \\ +2\tau_3\sqrt{\mu(t-s)};s)d\tau_1 d\tau_2 d\tau_3 ds - \dfrac{1}{2}\sqrt{\mu}\{\dfrac{1}{\sqrt{\pi^3}}\int_0^t\dfrac{1}{\sqrt{t-s}}[\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Phi_{0l_1^2}(x_1+ \\ +2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)d(x_1+2\tau_1\sqrt{\mu(t-s)})d\tau_2 d\tau_3 + \\ +\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Phi_{0l_2^2}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)\times \\ \times d\tau_1 d(x_2+2\tau_2\sqrt{\mu(t-s)})d\tau_3 + \int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Phi_{0l_3^2}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\times \\ \times\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)d\tau_1 d\tau_2 d(x_3+2\tau_3\sqrt{\mu(t-s)})]ds\} = \\ = \dfrac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\sum_{j=1}^{3}\sqrt{\mu}\,\dfrac{\tau_j}{\sqrt{t-s}}\Phi_{0l_j}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+ \\ +2\tau_3\sqrt{\mu(t-s)};s)d\tau_1 d\tau_2 d\tau_3 ds - \sqrt{\mu}\{\dfrac{1}{\sqrt{\pi^3}}\int_0^t\dfrac{1}{\sqrt{t-s}}\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\sum_{j=1}^{3}\tau_j\Phi_{0l_j}(x_1+ \\ +2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)d\tau_1 d\tau_2 d\tau_3 ds = 0. \end{cases}$$

That it was required to prove.

From the received results follows that functions υ_i are defined on the basis of (4.12), i.e.

$$\upsilon_i = \lambda_i[\vartheta_0(x_1,x_2,x_3) + H(x_1,x_2,x_3,t)], \forall (x_1,x_2,x_3,t) \in T, (i = \overline{1,3}). \tag{4.17}$$

Further, considering partial derivatives of 1st order systems (4.17) and summing up with acceptance in attention (1.2), (4.12) we have, that the system (4.17) satisfies to a condition (1.2):

$$\begin{cases} \sum_{i=1}^{3} \upsilon_{ix_i} = \sum_{i=1}^{3} \lambda_i \vartheta_{0x_i} + \sum_{i=1}^{3} \lambda_i H_{x_i} = \sum_{i=1}^{3} \lambda_i \vartheta_{0x_i} + \frac{1}{\sqrt{\pi^3}} \int_0^t \int_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2)) \{ \sum_{i=1}^{3} \lambda_i \Phi_{0l_i}(x_1 + 2\tau_1\sqrt{\mu(t-s)}, \\ x_2 + 2\tau_2\sqrt{\mu(t-s)}, x_3 + 2\tau_3\sqrt{\mu(t-s)}; s)\} d\tau_1 d\tau_2 d\tau_3 ds = 0, \\ l_i = x_i + 2\tau_i\sqrt{\mu(t-s)}, (i = \overline{1,3}), \\ \sum_{i=1}^{3} \lambda_i \vartheta_{0x_i} = 0; \quad \sum_{i=1}^{3} \lambda_i \Phi_{0x_i} = 0. \end{cases}$$

II. So as $\upsilon_{i0} \equiv \lambda_i \vartheta_0, (\vartheta_0 \in C^3(R^3))$, that solution of problem (1.1)-(1.3) belongs in $\tilde{C}^{3,1}_{n=3}(T)$:

$$\begin{cases} \|\nu\|_{\tilde{C}^{3,1}_{n=3}} = \sum_{i=1}^{3} \{ \sum_{0 \le |k| \le 3} \|D^k \upsilon_i\|_C + \|\upsilon_{it}\|_C \}; \quad \tilde{C}^{3,1}_{n=3}(T) \equiv \tilde{C}^{3,3,3,1}_{n=3}(T) \ne C^{3,3,3,1}_{n=3}(T), \\ \nu = (\upsilon_1, \upsilon_2, \upsilon_3); \quad \upsilon_i \equiv \lambda_i[\vartheta_0 + V], (i = \overline{1,3}), \vartheta_0 \in C^3(R^3), \Delta\vartheta_0 = 0, \\ \|Z\|_{\tilde{C}^{3,1}} = \sum_{0 \le |k| \le 3} \|D^k Z\|_C + \|Z_t\|_C, \\ k = 0: D^0 Z \equiv Z; \ k \ne 0: D^k Z = \frac{\partial^{|k|} Z}{\partial x_1^{\alpha_1} \partial x_2^{\alpha_2} \partial x_3^{\alpha_3}}; \quad |k| = \sum_{i=1}^{3} \alpha_i, (\alpha_i = \overline{0,3}). \end{cases}$$

Really, if

$$\begin{cases} \upsilon_{i0} \in C^3(R^3); \Phi_0: \ \sup_{R^3} |D^k \upsilon_{i0}| \le \gamma_1, \quad (i = \overline{1,3}), \\ \sup_T |D^k \Phi_0(x_1,x_2,x_3,t)| \le \beta_1, \quad (i = \overline{1,3}; \ k = \overline{0,3}), \\ \sup_T \frac{1}{\sqrt{\pi^3}} \int_0^t \int_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2)) |D^k \Phi_0(l_1,l_2,l_3;\tau)| d\tau_1 d\tau_2 d\tau_3 d\tau \le \beta_1 T_0 = \beta_2, \\ \sup_T \frac{1}{\sqrt{\pi^3}} \int_0^t \int_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2)) \frac{1}{\sqrt{t-\tau}} \sum_{j=1}^{3} |\tau_j| \times |\Phi^{(k)}_{0l_j^k}(l_1,l_2,l_3;\tau)| d\tau_1 d\tau_2 d\tau_3 d\tau \le \\ \le \beta_1 \frac{1}{\sqrt{\pi^3}} \sup_T \int_0^t \int_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2)) \frac{1}{\sqrt{t-\tau}} \sum_{j=1}^{3} |\tau_j| d\tau_1 d\tau_2 d\tau_3 d\tau \le 3\beta_1 \sqrt{2T_0} = \beta_3, \\ l_j = x_j + 2\tau_j \sqrt{\mu(t-\tau)}, (j = \overline{1,3}), \quad \beta = \max_{1 \le i \le 3}(\beta_i; \gamma_1), \quad \beta_0 = \beta(1 + \sqrt{\mu}), \end{cases} \tag{4.18}$$

that on a basis (4.15) we will receive

$$\|Z\|_{\tilde{C}^{3,1}} \le N_1 + \beta_0, \ (0 < N_1 = 20\beta).$$

In the conclusion estimating (4.17) it is had

$$\begin{cases} v=(\upsilon_1,\upsilon_2,\upsilon_3); \quad \upsilon_i=\lambda_i[\vartheta_0+Z],(i=\overline{1,3}): \\ \|v\|_{\tilde{C}^{3,1}_{n=3}(T)}=\sum_{i=1}^{3}[\|\lambda_i Z\|_{C^{3,0}(T)}+\|\lambda_i\vartheta_0\|_{C^3(T)}+\|\lambda_i Z_t\|_{C(T)}]\le d_0[N_1+N_2+\beta_0]=d_0[N_0+\beta_0], \\ \|Z\|_{\tilde{C}^{3,1}(T)}=\|Z\|_{C^{3,0}(T)}+\|Z_t\|_{C(T)}\le N_1+\beta_0, \\ d_0=\sum_{i=1}^{3}\lambda_i; \quad N_0=N_1+N_2=40\beta. \end{cases} \tag{4.19}$$

Lemma 2. In the conditions of (1.2), (4.11), (4.12) and (4.18) the equation (4.15) has a single solution in $\tilde{C}^{3,1}(T)$.

Theorem 4*. At performance of conditions of the lemma 2 the problem (1.1), (1.2), (4.11) has a smooth single solution in $\tilde{C}^{3,1}_{n=3}(T)$ of the defined by a rule (4.17).

Remarks: I. At performance of conditions of theorem 4* and

$$\begin{cases} \operatorname{rot} v\ne 0,\ (v=(\upsilon_1,\upsilon_2,\upsilon_3); \quad \upsilon_i=\lambda_i[\vartheta_0+Z];\ i=\overline{1,3}): \\ |\lambda_3\vartheta_{0x_2}(x_1,x_2,x_3)-\lambda_2\vartheta_{0x_3}(x_1,x_2,x_3)|\le h_1^0; \quad |\lambda_1\vartheta_{0x_3}(x_1,x_2,x_3)-\lambda_1\vartheta_{0x_1}(x_1,x_2,x_3)|\le h_2^0, \\ |\lambda_2\vartheta_{0x_1}(x_1,x_2,x_3)-\lambda_1\vartheta_{0x_2}(x_1,x_2,x_3)|\le h_3^0, \\ \sup_T \frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))|\lambda_3\Phi_{0l_2}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\times \\ \times\sqrt{\mu(t-s)};s)-\lambda_2\Phi_{0l_3}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)|d\tau_1 d\tau_2 d\tau_3 ds\le h_1, \\ \sup_T \frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))|\lambda_1\Phi_{0l_3}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\times \\ \times\sqrt{\mu(t-s)};s)-\lambda_3\Phi_{0l_1}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)|d\tau_1 d\tau_2 d\tau_3 ds\le h_2, \\ \sup_T \frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))|\lambda_2\Phi_{0l_1}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\times \\ \times\sqrt{\mu(t-s)};s)-\lambda_1\Phi_{0l_2}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)|d\tau_1 d\tau_2 d\tau_3 ds\le h_3, \\ l_i=x_i+2\tau_i\sqrt{\mu(t-s)},(i=\overline{1,3}), \end{cases}$$

takes place

$$\operatorname{rot} v\ne 0: \quad \sup_T|\operatorname{rot} v(x_1,x_2,x_3,t)|\le M_0<\infty, \ \left(\sum_{i=1}^{3}(h_i+h_i^0)=M_0\right). \tag{4.20}$$

In a consequence and (see (2.24)):

$$\sup_{R^3}\int_0^{T_0}|\operatorname{rot} v(x_1,x_2,x_3,t)|dt\le\sup_{R^3}\int_0^{T_0}\{|\frac{\partial}{\partial x_2}\upsilon_3(x_1,x_2,x_3,s)-\frac{\partial}{\partial x_3}\upsilon_2(x_1,x_2,x_3,s)|+|\frac{\partial}{\partial x_3}\upsilon_1(x_1,x_2,x_3,s)-$$
$$-\frac{\partial}{\partial x_1}\upsilon_3(x_1,x_2,x_3,s)|+|\frac{\partial}{\partial x_1}\upsilon_2(x_1,x_2,x_3,s)-\frac{\partial}{\partial x_2}\upsilon_1(x_1,x_2,x_3,s)|\}ds\le M_0T_0=M<\infty.$$

Well then, we obtain an estimation of type Beale-Kato-Majda [5].

II. Results 4.2 taking into account transformation (4.12) can be used concerning equations Navier-Stokes (1.1):

$$\frac{\partial \upsilon_i}{\partial t} + \sum_{j=1}^{3} \upsilon_j \frac{\partial \upsilon_i}{\partial x_j} = f_i - \frac{1}{\rho} P_{x_i} + \mu \Delta \upsilon_i, (i = \overline{1,3}), \tag{1}$$

and

$$\operatorname{div} \nu = 0, \forall (x,t) \in T = \overline{U}^3 \times [0, T_0], \tag{2}$$

in the limited area U^3 spaces of variables (x_1, x_2, x_3) the limited surface S, and for t, satisfying to an inequality $0 \le t \le T_0$, under conditions [13]:

$$\upsilon_i \big|_S = 0, \tag{3}$$

$$\upsilon_i \big|_{t=0} = \lambda_i \vartheta_0(x_1, x_2, x_3), (\vartheta_0 \big|_S = 0). \tag{4}$$

Really, applying transformation (4.12), i.e.:

$$\begin{cases} \upsilon_i = \lambda_i [\vartheta_0(x_1, x_2, x_3) + Z(x_1, x_2, x_3, t)], \forall (x_1, x_2, x_3, t) \in T = \overline{U}^3 \times [0, T_0], (i = \overline{1,3}), \\ Z \big|_S = 0, \quad \forall t \in [0, T_0], \\ Z \big|_{t=0} = 0, \ \forall (x_1, x_2, x_3) \in \overline{U}^3, \end{cases} \tag{5}$$

we will receive the equation (4.13), i.e.:

$$\lambda_i Z_t = f_i - \frac{1}{\rho} P_{x_i} + \mu \lambda_i \Delta Z, (i = \overline{1,3}). \tag{6}$$

Hence (4.14):

$$\begin{cases} \frac{1}{\rho} \Delta P = -4\pi F_0, \\ F_0 \equiv -\frac{1}{4\pi} \sum_{i=1}^{3} f_{ix_i}(x_1, x_2, x_3, t). \end{cases} \tag{7}$$

The equation (7) is the equation of Poisson and for descriptive reasons decisions we put a boundary problem in a kind:

$$\begin{cases} P \big|_S = \psi_1 \equiv 0, \quad \forall t \in [0, T_0], \\ \frac{\partial P}{\partial n} \Big|_S = \psi_2 \equiv 0. \end{cases} \tag{8}$$

Then equation (7) we obtain explicit representations for the unknown pressure P [13]:

$$\frac{1}{\rho} P = \int_{U^3} F_0(s_1, s_2, s_3, t) \frac{ds_1 ds_2 ds_3}{r} + \frac{1}{4\pi} \iint_S \frac{\partial \frac{1}{r}}{\partial n} \psi_1 dS - \frac{1}{4\pi} \iint_S \frac{1}{r} \psi_2 dS =$$
$$= \int_{U^3} F_0(s_1, s_2, s_3, t) \frac{ds_1 ds_2 ds_3}{r}, \ [\text{see.} (0.8): \ \psi_1, \psi_2 \equiv 0; \ r = \sqrt{\sum_{i=1}^{3} (x_i - s_i)^2}\,]. \tag{9}$$

Thus, considering (6), (9) we will receive [13]:

$$Z_t = \Phi_0 + \mu \Delta Z, \ \forall (x_1, x_2, x_3, t) \in T, \tag{10}$$

$$\begin{cases} Z\big|_S = 0, \ \forall t \in [0, T_0], \\ Z\big|_{t=0} = 0, \ \forall (x_1, x_2, x_3) \in \overline{U}^3, \end{cases} \tag{11}$$

where

$$(\lambda_1)^{-1}(f_1 - \rho^{-1} P_{x_1}) = (\lambda_2)^{-1}(f_2 - \rho^{-1} P_{x_2}) = (\lambda_3)^{-1}(f_3 - \rho^{-1} P_{x_3}) \equiv \Phi_0(x_1, x_2, x_3, t).$$

Full research of a problem (10), (11) is well shown in work [13, pp.349-351]. And it means that the generalised decision will be the decision in usual sense, at sufficient smoothness Φ_0 [13, pp.311-318, XXII]. Therefore similar conclusions we can make concerning a problem (1) - (4). That it was required to prove.

4.3. Modified Variant of a Method (4.12) with the Conditions $\mathrm{div} f = 0, \mathrm{rot}\, f = 0$

Let’s consider updating of the basic method 4.1. Here we will see that components of speed grow than in rules (4.2) or (4.12), when $0 < \mu < 1$, on the basis of regulatory functions Ω_i. Therefore in this case the problem (1.1)-(1.3) does not contain in itself restriction in kinds (A_1), (A_2) and in it urgency of research in a case (A_3) and $\mathrm{div} f = 0, \ \mathrm{rot}\, f = 0$.

Offered methods of integrated transformations are entered so that to transform nonlinear problems of Navier-Stokes to linear problems of heat conductivity [13]. Questions solvability of the problem are proved on the basis of the developed methods. These methods are initial problems lead to the integral equations where possibility to find the analytical solution taking into account the theory of the integral equations of Volterra-Abel [13].

For incompressible currents with a friction, when

$$\begin{cases} \upsilon_i\big|_{t=0} = \upsilon_{i0}(x_1, x_2, x_3) \equiv \lambda_i \vartheta_0(x_1, x_2, x_3), (i = \overline{1,3}), \\ \sum_{i=1}^{3} \lambda_i \vartheta_{0 x_i} = 0; \ \ \Delta \vartheta_0 = 0; \ \vartheta_0 \in R^3; \ \mathrm{div} f = 0, \ \mathrm{rot}\, f = 0, (\Delta f_i = 0; \ i = \overline{1,3}), \\ \sup_T \left| D^k f_i \right| \le N_0 = \mathrm{const}, \ (T = R^3 \times [0, T_0]), \\ \Omega_i(x_1, x_2, x_3, t) \equiv \mu \int_0^t f_i(x_1, x_2, x_3, s) ds, \ \ (i = \overline{1,3}), \end{cases} \tag{4.21}$$

functions $\upsilon_i, i = 1,2,3$ is represented in a kind

$$\begin{cases} \upsilon_i = \lambda_i [\vartheta_0(x_1, x_2, x_3) + Z(x_1, x_2, x_3, t)] + \Omega_i(x_1, x_2, x_3, t), \forall (x_1, x_2, x_3, t) \in T, (i = \overline{1,3}), \\ Z\big|_{t=0} = 0, \ \ \forall (x_1, x_2, x_3) \in R^3, \end{cases} \tag{4.22}$$

where $0 < \lambda_i -$ the known constants.

From the entered functions Ω_i, is required

$$\begin{cases} \Omega_{it} \equiv \mu f_i(x_1,x_2,x_3,t), (i=\overline{1,3}), \\ \Delta f_i = 0: \quad \Delta\Omega_i = 0, \quad (\left|D^k\Omega_i\right| \equiv \left|\mu\int_0^t D^k f_i(x_1,x_2,x_3,t,s)ds\right| \le N_0\mu T_0; \quad i=\overline{1,3}), \end{cases}$$

at that all functions Ω_i are regular concerning viscosity parameter $(0,1)\in\mu$, (here μ in a role of small parameter),

$$\lim_{\mu\to 0}\Omega_i(x_1,x_2,x_3,t) = 0, \forall(x_1,x_2,x_3,t)\in T.$$

Further, supposing (4.21), (4.22) and

$$\begin{cases} \operatorname{div} v = 0: \sum_{i=1}^{3}\lambda_i Z_{x_i} = 0; \ \sum_{i=1}^{3}\lambda_i\vartheta_{0x_i} = 0; \ \sum_{i=1}^{3}\Omega_{ix_i} = 0, \\ U = \sum_{j=1}^{3}\Omega_j^2; \ \sum_{j=1}^{3}\Omega_j\Omega_{ix_j} = \frac{1}{2}(\sum_{j=1}^{3}\Omega_j^2)_{x_i} = \frac{1}{2}U_{x_i}, \\ \sum_{j=1}^{3}\upsilon_j\upsilon_{ix_j} \equiv \sum_{j=1}^{3}\lambda_j(\vartheta_0+Z)\Omega_{ix_j} + \sum_{j=1}^{3}\Omega_j\lambda_i(\vartheta_{0x_j}+Z_{x_j}) + \frac{1}{2}U_{x_i}, \\ \sum_{j=1}^{3}\lambda_j(\vartheta_0+Z)\lambda_i(\vartheta_{0x_j}+Z_{x_j}) = \lambda_i(\vartheta_0+Z)\sum_{j=1}^{3}\lambda_j(\vartheta_{0x_j}+Z_{x_j}) = 0, \\ \upsilon_{it} \equiv \lambda_i Z_t + \Omega_{it}, \ (i=\overline{1,3}), \\ \mu\Delta\upsilon_i \equiv \mu[\lambda_i(\Delta\vartheta_0+\Delta Z)+\Delta\Omega_i] = \mu\lambda_i\Delta Z, (i=\overline{1,3}; \ \Delta\vartheta_0 = 0), \end{cases} \tag{4.23}$$

for incompressible currents with a friction the equations of Navier-Stokes (1.1) become simpler as take place (4.21)-(4.23). Therefore the problem (1.1) - (1.3), is led to a kind

$$\lambda_i Z_t + \sum_{j=1}^{3}\lambda_j(\vartheta_0+Z)\Omega_{ix_j} + \sum_{j=1}^{3}\Omega_j\lambda_i(\vartheta_{0x_j}+Z_{x_j}) + \frac{1}{2}U_{x_i} = (1-\mu)f_i - \frac{1}{\rho}P_{x_i} + \mu\lambda_i\Delta Z, (i=\overline{1,3}). \tag{4.24}$$

The system (1.1) is equivalent transformed to a kind (4.24). Then a new system of equations is called of inhomogeneous linear system a transfer of vortices [12]. Here the inertial terms in the equations (1.1) are linearized using regulatory functions Ω_i, which were first introduced in [8] and in the method (4.22).

From system (4.24), considering conditions (4.21)-(4.23) and having entered [8] APS we have the equation:

$$\begin{cases} \sum_{i=1}^{3}\frac{\partial}{\partial x_i}(4.24): \quad \Delta(\frac{1}{\rho}P + \frac{1}{2}U) = -\{F_0 + B[Z_{x_1},Z_{x_2},Z_{x_3}]\}, \\ (B[Z_{x_1},Z_{x_2},Z_{x_3}])(x_1,x_2,x_3,t) \equiv \sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j\Omega_{ix_j})Z_{x_i} + \sum_{i=1}^{3}(\sum_{j=1}^{3}\Omega_{jx_i}Z_{x_j})\lambda_i, \\ \operatorname{div} f = 0; \quad F_0(x_1,x_2,x_3,t) \equiv \sum_{i=1}^{3}\lambda_i(\sum_{j=1}^{3}\Omega_{jx_i}\vartheta_{0x_j}) + \sum_{i=1}^{3}\vartheta_{0x_i}(\sum_{j=1}^{3}\lambda_j\Omega_{ix_j}), \end{cases}$$

$$\begin{cases} \frac{1}{\rho}P + \frac{1}{2}U = \frac{1}{4\pi}\int_{R^3}\frac{1}{r}\{F_0(s_1,s_2,s_3,t) + (B[Z_{s_1},Z_{s_2},Z_{s_3}])(s_1,s_2,s_3,t)\}ds_1ds_2ds_3, \\ \frac{1}{\rho}P_{x_i} + \frac{1}{2}U_{x_i} = \frac{1}{4\pi}\int_{R^3}\frac{\tau_i}{\sqrt{(\tau_1^2+\tau_2^2+\tau_3^2)^3}}\{F_0(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t) + \\ +(B[Z_{h_1},Z_{h_2},Z_{h_3}])(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t)\}d\tau_1d\tau_2d\tau_3,\ (s_i - x_i = \tau_i; h_i = x_i+\tau_i; i=\overline{1,3}), \end{cases} \quad (4.25)$$

so as takes place

$$\begin{cases} \sum_{i=1}^{3}\frac{\partial}{\partial x_i}\{\sum_{j=1}^{3}\lambda_j(\vartheta_0 + Z)\Omega_{ix_j} + \sum_{j=1}^{3}\Omega_j\lambda_i(\vartheta_{0x_j} + Z_{x_j})\} = \sum_{i=1}^{3}\lambda_i(\sum_{j=1}^{3}\Omega_{jx_i}\vartheta_{0x_j}) + \sum_{i=1}^{3}\vartheta_{0x_i}(\sum_{j=1}^{3}\lambda_j\Omega_{ix_j}) + \\ +\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j\Omega_{ix_j})Z_{x_i} + \sum_{i=1}^{3}(\sum_{j=1}^{3}\Omega_{jx_i}Z_{x_j})\lambda_i, \\ (\sum_{j=1}^{3}\Omega_{jx_j})_{x_i} = 0;\ (\sum_{j=1}^{3}\lambda_j\vartheta_{0x_j})_{x_i} = 0,\ (\sum_{j=1}^{3}\lambda_jZ_{x_j})_{x_i} = 0, (i=\overline{1,3}), \\ \frac{\partial}{\partial t}[\sum_{i=1}^{3}\lambda_iZ_{x_i}] = 0;\ \mu\sum_{i=1}^{3}\frac{\partial}{\partial x_i}(\lambda_i\Delta Z) = 0, \\ \sum_{i=1}^{3}\frac{\partial}{\partial x_i}(\frac{1}{2}U_{x_i}) = \frac{1}{2}\Delta U;\quad \sum_{i=1}^{3}\frac{\partial}{\partial x_i}(-\frac{1}{\rho}P_{x_i}) \equiv -\frac{1}{\rho}\Delta P, \\ \lim_{\mu\to 0}B[Z_{x_1},Z_{x_2},Z_{x_3}] = 0, \forall(x_1,x_2,x_3,t)\in T. \end{cases}$$

There are various methods [12] which give communication of speed and pressure. For example, if the law of the pressure distribution obtained from the Bernoulli equation. Here is received the new law of distribution of pressure in the form of (4.25), for the first time similar results are received in work [8]. Then on a basis (4.25) system (4.24) it is equivalent, will be transformed to a kind:

$$\begin{cases} Z_t = \Phi_0 + (Q[Z,Z_{x_1},Z_{x_2},Z_{x_3}])(x_1,x_2,x_3,t) + \mu\Delta Z, (i=\overline{1,3}), \\ Z\big|_{t=0} = 0, \forall(x_1,x_2,x_3)\in R^3, \end{cases} \quad (4.26)$$

where

$$\begin{cases} \Phi_0(x_1,x_2,x_3,t) \equiv -\sum_{j=1}^{3}\Omega_j\vartheta_{0x_j} + d_0^{-1}[\sum_{i=1}^{3}(1-\mu)f_i - \sum_{i=1}^{3}\vartheta_0(\sum_{j=1}^{3}\lambda_j\Omega_{ix_j}) - \\ -\frac{1}{4\pi}\int_{R^3}\{\sum_{i=1}^{3}\frac{\tau_i}{\sqrt{(\tau_1^2+\tau_2^2+\tau_3^2)^3}}(F_0(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t))\}d\tau_1d\tau_2d\tau_3];\ d_0 = \sum_{i=1}^{3}\lambda_i > 0, \\ (Q[Z,Z_{x_1},Z_{x_2},Z_{x_3}])(x_1,x_2,x_3,t) \equiv -\{d_0^{-1}Z(x_1,x_2,x_3,t)[\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j\Omega_{ix_j}(x_1,x_2,x_3,t))] + \\ +\sum_{j=1}^{3}Z_{x_j}(x_1,x_2,x_3,t)\Omega_j(x_1,x_2,x_3,t) + d_0^{-1}[\frac{1}{4\pi}\int_{R^3}(\sum_{i=1}^{3}\frac{\overline{\tau}_i}{\sqrt{(\overline{\tau}_1^2+\overline{\tau}_2^2+\overline{\tau}_3^2)^3}}(B[Z_{\overline{h}_1},Z_{\overline{h}_2},Z_{\overline{h}_3}])(x_1 + \\ +\overline{\tau}_1,x_2+\overline{\tau}_2,x_3+\overline{\tau}_3;t))d\overline{\tau}_1d\overline{\tau}_2d\overline{\tau}_3]\}, \\ \overline{h}_i = x_i + \overline{\tau}_i, (i=\overline{1,3}). \end{cases}$$

The problem (4.26) is led to system of the integrated equations quite regular rather $\mu \in (0,1)$, in a kind

$$\begin{cases} Z_{x_i} = W_i(x_1,x_2,x_3,t), \forall (x_1,x_2,x_3,t) \in T, (i=\overline{1,3}), \\ Z = M_1 + \frac{1}{8\sqrt{\pi^3}} \int_0^t \int_{R^3} \exp(-\frac{r^2}{4\mu(t-s)}) \frac{1}{(\sqrt{\mu(t-s)})^3} (Q[Z,W_1,W_2,W_3])(s_1,s_2,s_3,s) \times \\ \times ds_1 ds_2 ds_3 ds = M_1 + \frac{1}{\sqrt{\pi^3}} \int_0^t \int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2))(Q[Z,W_1,W_2,W_3])(x_1+2\tau_1\sqrt{\mu(t-s)}, x_2 + \\ +2\tau_2\sqrt{\mu(t-s)}, x_3 + 2\tau_3\sqrt{\mu(t-s)}; s) d\tau_1 d\tau_2 d\tau_3 ds \equiv (\Psi_0[Z,W_1,W_2,W_3])(x_1,x_2,x_3,t), \\ W_i = M_{1x_i} + \frac{1}{8\sqrt{\pi^3}} \int_0^t \int_{R^3} (\exp(-\frac{r^2}{4\mu(t-s)})) \frac{-(x_i-s_i)}{2\mu(t-s)} \frac{1}{(\sqrt{\mu(t-s)})^3} (Q[Z,W_1,W_2,W_3]) \times \\ \times (s_1,s_2,s_3,s) ds_1 ds_2 ds_3 ds = M_{1x_i} + \frac{1}{\sqrt{\pi^3}} \int_0^t \int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)) \frac{\tau_i}{\sqrt{\mu(t-s)}} \times \\ \times (Q[Z,W_1,W_2,W_3])(x_1+2\tau_1\sqrt{\mu(t-s)}, x_2+2\tau_2\sqrt{\mu(t-s)}, x_3+2\tau_3\sqrt{\mu(t-s)}; s) d\tau_1 d\tau_2 d\tau_3 ds \equiv \\ \equiv (\Psi_i[Z,W_1,W_2,W_3])(x_1,x_2,x_3,t), \quad (s_i - x_i = 2\tau_i\sqrt{\mu(t-s)}; i=\overline{1,3}), \\ M_1(x_1,x_2,x_3,t) \equiv \frac{1}{\sqrt{\pi^3}} \int_0^t \int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)) \Phi_0(x_1+2\tau_1\sqrt{\mu(t-s)}, x_2+2\tau_2\sqrt{\mu(t-s)}, x_3 + \\ +2\tau_3\sqrt{\mu(t-s)}; s) d\tau_1 d\tau_2 d\tau_3 ds. \end{cases} \tag{4.27}$$

The received system (4.27) consists of four integral equations, i.e. Volterra and Volterra-Abel on a variable $t \in [0,T_0]$ and thus contains in itself of four unknown functions. Therefore there is no necessity to think out various algorithms for the decision of this system and it is enough to show conditions which provide contraction mapping principle and for the decision of this system to use a of Picard's method.

Therefore, if takes place:

$$\begin{cases} \forall (x_1,x_2,x_3,t) \in T; M_1, \Pi, \Omega_i : \\ \sup_T \left| D^k M_1(x_1,x_2,x_3,t) \right| \le \beta_1, \ (k=\overline{0,3}; \ \ t-s=\tau), \\ \sup_{T\times T} \Pi(x_1,x_2,x_3,\tau_1,\tau_2,\tau_3;t,\tau) \equiv \sup_{T\times T} \{ d_0^{-1} \sum_{i=1}^3 (\sum_{j=1}^3 \lambda_j \left| \Omega_{is_j}(x_1+2\tau_1\sqrt{\mu\tau}, x_2+2\tau_2\sqrt{\mu\tau}, x_3+2\tau_3 \times \right. \\ \left. \times\sqrt{\mu\tau}; t-\tau) \right|) + \sum_{j=1}^3 \left| \Omega_j(x_1+2\tau_1\sqrt{\mu\tau}, x_2+2\tau_2\sqrt{\mu\tau}, x_3+2\tau_3\sqrt{\mu\tau}; t-\tau) \right| + \\ + d_0^{-1} [\frac{1}{4\pi} \int_{R^3} (\sum_{i=1}^3 \frac{\left|\bar{\tau}_i\right|}{\sqrt{(\bar{\tau}_1^2+\bar{\tau}_2^2+\bar{\tau}_3^2)^3}} \{ \sum_{i=1}^3 (\sum_{j=1}^3 \lambda_j \left| \Omega_{il_j}(x_1+2\tau_1\sqrt{\mu\tau}+\bar{\tau}_1, x_2+2\tau_2\sqrt{\mu\tau}+\bar{\tau}_2, x_3 + \right. \\ \left. +2\tau_3\sqrt{\mu\tau}+\bar{\tau}_3; t-\tau) \right|) + \sum_{i=1}^3 (\sum_{j=1}^3 \left| \Omega_{jl_i}(x_1+2\tau_1\sqrt{\mu\tau}+\bar{\tau}_1, x_2+2\tau_2\sqrt{\mu\tau}+\bar{\tau}_2, x_3+2\tau_3\sqrt{\mu\tau} + \right. \\ \left. +\bar{\tau}_3; t-\tau) \right|) \lambda_i \}) d\bar{\tau}_1 d\bar{\tau}_2 d\bar{\tau}_3] \} \le \beta_2 \mu, \end{cases}$$

$$\begin{cases} k_0 = \dfrac{1}{\sqrt{\pi^3}} \sup_T \int_0^t \int_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2)) \Pi(x_1, x_2, x_3, \tau_1, \tau_2, \tau_3; t, \tau) d\tau_1 d\tau_2 d\tau_3 d\tau \le \beta_2 \mu T_0, \\ k_i = \dfrac{1}{\sqrt{\pi^3}} \sup_T \int_0^t \int_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2)) \Pi(x_1, x_2, x_3, \tau_1, \tau_2, \tau_3; t, \tau) \dfrac{|\tau_i|}{\sqrt{\mu \tau}} d\tau_1 d\tau_2 d\tau_3 d\tau \le \\ \le \sqrt{2T_0} \beta_2 \sqrt{\mu}, (i = \overline{1,3}), \\ \beta = \max(\beta_2 T_0;\ 3\sqrt{2T_0} \beta_2), \end{cases} \quad (4.28)$$

and if operators: $\Psi_i, (i = \overline{0,3})$ compressing with a compression factor k_i,

$$\begin{cases} \Psi_i, (i = \overline{0,3}):\ k_i \le \dfrac{h}{4}, (h < 1),\ (i = \overline{1,3}), \\ \sum_{i=0}^{3} k_i \le \sqrt{\mu}(\beta_2 T_0 \sqrt{\mu} + 3\sqrt{2T_0} \beta_2) \le \sqrt{\mu}(\sqrt{\mu} + 1)\beta = h < 1, \\ S_{r_1}(0) = \{Z, W_i : |V|;\ |W_i| \le r_1, \forall (x_1, x_2, x_3, t) \in T\}, (i = \overline{1,3}), \end{cases} \quad (4.29)$$

and:

$$\begin{cases} \|\Psi_i[0,0,0,0]\|_C \le r_1(1-h): \\ \|\Psi_i[Z, W_1, W_2, W_3]\|_C \le \|\Psi_i[Z, W_1, W_2, W_3] - \Psi_i[0,0,0,0]\|_C + \|\Psi_i[0,0,0,0]\|_C \le \\ \le k_i 4 r_1 + r_1(1-h) \le h r_1 + r_1(1-h) = r_1, \\ \Psi_i : S_{r_1}(0) \to S_{r_1}(0),\ (i = \overline{0,3}). \end{cases} \quad (4.30)$$

Then on the basis of a contraction mapping principle system (4.27) is solvable and solution of this system we can find on the basis of Picard's method

$$\begin{cases} \forall (x_1, x_2, x_3, t) \in T:\quad Z_{n+1} = \Psi_0[Z_n, W_{1,n}, W_{2,n}, W_{3,n}], \\ W_{i,n+1} = \Psi_i[Z_n, W_{1,n}, W_{2,n}, W_{3,n}], (n = 0, 1, \ldots; Z_0 = 0;\ W_{i,0} = 0;\ i = \overline{1,3}), \end{cases} \quad (4.31)$$

at that

$$\begin{cases} E_{n+1} = \|Z_{n+1} - Z_n\|_C + \sum_{i=1}^{3} \|W_{i,n+1} - W_{i,n}\|_C;\ E_n = \|Z_n - Z_{n-1}\|_C + \sum_{i=1}^{3} \|W_{i,n} - W_{i,n-1}\|_C: \\ \|Z_{n+1} - Z_n\|_C \le k_0 E_n;\quad \|W_{i,n+1} - W_{i,n}\|_C \le k_i E_n, (i = \overline{1,3}), \\ E_{n+1} \le h E_n \le \ldots \le h^n E_1 \xrightarrow[n \to \infty]{h<1} 0, \\ \|Z_{n+k} - Z_n\|_C \le \sum_{j=0}^{k-1} k_0 E_{n+j};\quad \|W_{i,n+k} - W_{i,n}\|_C \le \sum_{j=0}^{k-1} k_i E_{n+j}, (i = \overline{1,3}), \\ E_{n+k} \le h \sum_{j=0}^{k-1} E_{n+j} \le \ldots \le h \sum_{j=0}^{k-1} h^{n+j-1} E_1 \le E_1 h^n \sum_{j=0}^{k-1} h^j \le E_1 h^n \dfrac{1}{1-h} \xrightarrow[n \to \infty]{h<1} 0, \\ U_0 = \|Z\|_C + \sum_{i=1}^{3} \|W_i\|_C;\ U_{n+1} \equiv \|Z_{n+1} - Z\|_C + \sum_{i=1}^{3} \|W_{i,n+1} - W_i\|_C, \\ U_{n+1} \le h^{n+1} U_0 \xrightarrow[n \to \infty]{h<1} 0, \\ Z_{n+1} \xrightarrow[n \to \infty]{h<1} Z \equiv H \in \tilde{C}^{3,1}(T);\quad W_{i,n+1} \xrightarrow[n \to \infty]{h<1} W_i, \forall (x_1, x_2, x_3, t) \in T, (i = \overline{1,3}). \end{cases} \quad (4.32)$$

Hence on the basis of (4.22)

$$\begin{cases} \upsilon_{i,n+1} = \lambda_i[\vartheta_0(x_1,x_2,x_3) + Z_{n+1}(x_1,x_2,x_3,t)] + \Omega_i(x_1,x_2,x_3,t),(n = 0,1,2,\ldots; i = \overline{1,3}), \\ \left\|\upsilon_{i,n+1} - \upsilon_i\right\|_C \le \lambda_i \left\|Z_{n+1} - Z\right\|_C \le \lambda_i h U_n \le \lambda_i h^{n+1} U_0 \xrightarrow[n\to\infty]{h<1} 0,(i = \overline{1,3}). \end{cases} \quad (4.33)$$

And it means that sequence $\{\upsilon_{i,n}\}_0^\infty$ converging to a limit $\upsilon_i,(i = \overline{1,3}):$

$$\upsilon_{i,n+1} \xrightarrow[n\to\infty]{h<1} \upsilon_i \in \tilde{C}^{3,1}(T),(i = \overline{1,3}). \quad (4.34)$$

Theorem 5*. Under conditions (1.2), (1.3), (4.21), (4.22), (4.28)-(4.30) and (4.34) problem Navier-Stokes has the smooth single solution in $\tilde{C}^{3,1}_{n=3}(T)$.

Remark 2. If takes place $0 < \beta \le 2^{-1}$, that $0 < \mu < 1$. In a case $\beta > 2^{-1}$, then

$$0 < \mu < (2^{-1}[\sqrt{1 + 4\beta^{-1}} - 1])^2 < 1. \quad (4.35)$$

4.4. Updating of a Method (4.2), When $\mathrm{div} f = 0,\ \mathrm{rot}\, f \ne 0,\ \Delta\vartheta_0 \ne 0$

Let consider updating of the basic method of paragraph 4.1: (4.2), when:

$$\begin{cases} \upsilon_i\big|_{t=0} = \upsilon_{i0}(x_1,x_2,x_3) \equiv \lambda_i\vartheta_0(x_1,x_2,x_3),(i = \overline{1,3}), \\ \vartheta_0 \in R^3;\ \ \Delta\vartheta_0 \ne 0;\ \ \sum_{i=1}^{3} \lambda_i \vartheta_{0x_i} = 0, \\ f_i \equiv \varphi_i + \overline{\Omega}_{it};\ \ \mathrm{div} f = 0:\ \mathrm{div}\varphi = 0;\ \ \sum_{i=1}^{3} \frac{\partial}{\partial x_i}\overline{\Omega}_{it} = 0, \\ \overline{\Omega}_{it} = \mu\, \overline{f}_i(x_1,x_2,x_3,t),\ (i = \overline{1,3}),\ \ \varphi = (\varphi_1,\varphi_2,\varphi_3);\ \overline{f} = (\overline{f}_1, \overline{f}_2, \overline{f}_3), \\ \mathrm{div}\overline{f} = 0;\ \mathrm{rot}\,\overline{f} = 0:\ \Delta\overline{f}_i = 0,(\Delta\overline{\Omega}_i = 0), \\ U = \sum_{j=1}^{3} \overline{\Omega}_j^2;\ \ \sum_{j=1}^{3} \overline{\Omega}_j\overline{\Omega}_{ix_j} = \frac{1}{2}(\sum_{j=1}^{3}\overline{\Omega}_j^2)_{x_i} = \frac{1}{2}U_{x_i}, \\ \overline{\Omega}_i(x_1,x_2,x_3,t) \equiv \mu\int_0^t \overline{f}_i(x_1,x_2,x_3,s')ds',(i = \overline{1,3}). \end{cases} \quad (4.36)$$

Then functions $\upsilon_i, i = \overline{1,3}$ is represented in a kind

$$\begin{cases} \upsilon_i = \lambda_i V(x_1,x_2,x_3,t) + \overline{\Omega}_i(x_1,x_2,x_3,t),(i = \overline{1,3}), \\ V\big|_{t=0} = \vartheta_0(x_1,x_2,x_3),\forall(x_1,x_2,x_3) \in R^3, \\ \mathrm{div}\, v = 0: \\ \sum_{i=1}^{3} \lambda_i V_{x_i} = 0;\ \ \sum_{i=1}^{3} \overline{\Omega}_{ix_i} \equiv \mu\int_0^t (\sum_{i=1}^{3} \overline{f}_{ix_i}(x_1,x_2,x_3,s))ds = 0, \end{cases} \quad (4.37)$$

where $0 < \lambda_i -$ the known constants.

Further, supposing (4.36), (4.37) and

$$\begin{cases} \sum_{j=1}^{3} \upsilon_j \upsilon_{ix_j} \equiv V \sum_{j=1}^{3} \lambda_j \overline{\Omega}_{ix_j} + \lambda_i \sum_{j=1}^{3} V_{x_j} \overline{\Omega}_j + \frac{1}{2} \overline{U}_{x_i}, (\lambda_i V \sum_{j=1}^{3} \lambda_j V_{x_j} = 0), \\ \upsilon_{it} \equiv \lambda_i V_t + \overline{\Omega}_{it}, \quad \mu \Delta \upsilon_i \equiv \mu[\lambda_i \Delta V + \Delta \overline{\Omega}_i] = \mu \lambda_i \Delta V, (i = \overline{1,3}; \quad \Delta \overline{\Omega}_i = 0), \end{cases} \tag{4.38}$$

for incompressible currents with a friction the equations of Navier-Stokes (1.1) become simpler as take place (4.36)-(4.38). Therefore the problem (1.1)-(1.3), is led to a kind

$$\lambda_i V_t + V \sum_{j=1}^{3} \lambda_j \overline{\Omega}_{ix_j} + \lambda_i \sum_{j=1}^{3} V_{x_j} \overline{\Omega}_j + \frac{1}{2} \overline{U}_{x_i} = \varphi_i - \frac{1}{\rho} P_{x_i} + \mu \lambda_i \Delta V, (i = \overline{1,3}). \tag{4.39}$$

From system (4.39), considering conditions (4.36)-(4.38) and having entered APS we have the equation:

$$\begin{cases} \sum_{i=1}^{3} \frac{\partial}{\partial x_i}(4.39): \quad \Delta(\frac{1}{\rho} P + \frac{1}{2} \overline{U}) = -\overline{B}[V_{x_1}, V_{x_2}, V_{x_3}], \\ (\overline{B}[V_{x_1}, V_{x_2}, V_{x_3}])(x_1, x_2, x_3, t) \equiv \sum_{i=1}^{3} V_{x_i} (\sum_{j=1}^{3} \lambda_j \overline{\Omega}_{ix_j}) + \sum_{i=1}^{3} (\sum_{j=1}^{3} \overline{\Omega}_{jx_i} V_{x_j}) \lambda_i, \\ \frac{1}{\rho} P + \frac{1}{2} \overline{U} = \frac{1}{4\pi} \int_{R^3} \frac{1}{r} \{(\overline{B}[V_{s_1}, V_{s_2}, V_{s_3}])(s_1, s_2, s_3, t)\} ds_1 ds_2 ds_3, \\ \frac{1}{\rho} P_{x_i} + \frac{1}{2} \overline{U}_{x_i} = \frac{1}{4\pi} \int_{R^3} \frac{\tau_i}{\sqrt{(\tau_1^2 + \tau_2^2 + \tau_3^2)^3}} \{(\overline{B}[V_{h_1}, V_{h_2}, V_{h_3}])(x_1 + \tau_1, x_2 + \tau_2, x_3 + \\ + \tau_3; t)\} d\tau_1 d\tau_2 d\tau_3, \\ s_i - x_i = \tau_i; h_i = x_i + \tau_i, (i = \overline{1,3}), \end{cases} \tag{4.40}$$

so as takes place

$$\begin{cases} \frac{\partial}{\partial t}[\sum_{i=1}^{3} \lambda_i V_{x_i}] = 0; \ \operatorname{div} \varphi = 0, \quad \mu \sum_{i=1}^{3} \frac{\partial}{\partial x_i}(\lambda_i \Delta V) = 0, \\ \sum_{i=1}^{3} \frac{\partial}{\partial x_i}(\frac{1}{2} \overline{U}_{x_i}) \equiv \frac{1}{2} \Delta \overline{U}; \quad \sum_{i=1}^{3} \frac{\partial}{\partial x_i}(-\frac{1}{\rho} P_{x_i}) \equiv -\frac{1}{\rho} \Delta P, \\ \sum_{i=1}^{3} \frac{\partial}{\partial x_i}(V \sum_{j=1}^{3} \lambda_j \overline{\Omega}_{ix_j}) \equiv \sum_{i=1}^{3} \lambda_i (\sum_{j=1}^{3} V_{x_j} \overline{\Omega}_{jx_i}) + \sum_{j=1}^{3} \overline{\Omega}_j \frac{\partial}{\partial x_j}(\sum_{i=1}^{3} \lambda_i V_{x_i})_{x_j} = \sum_{i=1}^{3} \lambda_i (\sum_{j=1}^{3} V_{x_j} \overline{\Omega}_{jx_i}), \\ \sum_{i=1}^{3} \frac{\partial}{\partial x_i}(\lambda_i \sum_{j=1}^{3} V_{x_j} \overline{\Omega}_j) \equiv \sum_{i=1}^{3} V_{x_i} (\sum_{j=1}^{3} \lambda_j \overline{\Omega}_{ix_j}) + V \sum_{j=1}^{3} \lambda_j (\sum_{i=1}^{3} \overline{\Omega}_{ix_i})_{x_j} = \sum_{i=1}^{3} V_{x_i} (\sum_{j=1}^{3} \lambda_j \overline{\Omega}_{ix_j}), \\ \sum_{j=1}^{3} \overline{\Omega}_j \frac{\partial}{\partial x_j}(\sum_{i=1}^{3} \lambda_i V_{x_i})_{x_j} = 0; \quad V \sum_{j=1}^{3} \lambda_j (\sum_{i=1}^{3} \overline{\Omega}_{ix_i})_{x_j} = 0, \\ \lim_{\mu \to 0} \overline{B}[V_{x_1}, V_{x_2}, V_{x_3}] = 0. \end{cases}$$

Then on a basis (4.40) system (4.39) it is equivalent, will be transformed to a kind:

$$\begin{cases} V_t + d^{-1}V[\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j\overline{\Omega}_{ix_j})] + \sum_{j=1}^{3}V_{x_j}\overline{\Omega}_j = \Phi_0 - d^{-1}[\frac{1}{4\pi}\int_{R^3}(\sum_{i=1}^{3}\frac{\tau_i}{\sqrt{(\tau_1^2+\tau_2^2+\tau_3^2)^3}}\times \\ \times\{(\overline{B}[V_{h_1},V_{h_2},V_{h_3}])(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t)\})d\tau_1 d\tau_2 d\tau_3] + \mu\Delta V, (i=\overline{1,3}), \\ d = \sum_{i=1}^{3}\lambda_i > 0; \quad \Phi_0(x_1,x_2,x_3,t) \equiv d^{-1}\sum_{i=1}^{3}\varphi_i(x_1,x_2,x_3,t), \end{cases} \tag{4.41}$$

or from (4.41), follows:

$$\begin{cases} V = M_1(x_1,x_2,x_3,t) + \frac{1}{8\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-\frac{r^2}{4\mu(t-s)})\frac{1}{(\sqrt{\mu(t-s)})^3}\times \\ \times(Q[V,W_1,W_2,W_3])(s_1,s_2,s_3,s)ds_1ds_2ds_3ds = M_1 + \frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\times \\ \times(Q[V,W_1,W_2,W_3])(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)\times \\ \times d\tau_1d\tau_2d\tau_3ds \equiv \Psi_0[V,W_1,W_2,W_3], \\ W_i = M_{1x_i} + \frac{1}{8\sqrt{\pi^3}}\int_0^t\int_{R^3}\frac{-(x_i-s_i)}{2\mu(t-s)}(\exp(-\frac{r^2}{4\mu(t-s)}))\frac{1}{(\sqrt{\mu(t-s)})^3}\times \\ \times(Q[V,W_1,W_2,W_3])(s_1,s_2,s_3,s)ds_1ds_2ds_3ds = M_{1x_i} + \frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\times \\ \times\frac{\tau_i}{\sqrt{\mu(t-s)}}(Q[V,W_1,W_2,W_3])(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\times \\ \times\sqrt{\mu(t-s)};s)d\tau_1d\tau_2d\tau_3ds \equiv \Psi_i[V,W_1,W_2,W_3], \end{cases} \tag{4.42}$$

where

$$\begin{cases} V_{x_i} = W_i(x_1,x_2,x_3,t), \forall(x_1,x_2,x_3,t)\in T, (i=\overline{1,3}), \\ M_1 \equiv \frac{1}{\sqrt{\pi^3}}\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\vartheta_0(x_1+2\tau_1\sqrt{\mu t},x_2+2\tau_2\sqrt{\mu t},x_3+2\tau_3\sqrt{\mu t})d\tau_1d\tau_2d\tau_3 + \\ + \frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Phi_0(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\times \\ \times\sqrt{\mu(t-s)};s)d\tau_1d\tau_2d\tau_3ds, \\ s_j - x_j = 2\tau_j\sqrt{\mu t}; \quad s_j - x_j = 2\tau_j\sqrt{\mu(t-s)}, (j=\overline{1,3}), \\ (Q[V,V_{s_1},V_{s_2},V_{s_3}])(s_1,s_2,s_3,s) \equiv -\{d^{-1}[V(s_1,s_2,s_3,s)\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j\overline{\Omega}_{is_j}(s_1,s_2,s_3,s))] + \\ + \sum_{j=1}^{3}V_{s_j}(s_1,s_2,s_3,s)\overline{\Omega}_j(s_1,s_2,s_3,s) + d^{-1}[\frac{1}{4\pi}\int_{R^3}(\sum_{i=1}^{3}\overline{\tau}_i\frac{1}{\sqrt{(\overline{\tau}_1^2+\overline{\tau}_2^2+\overline{\tau}_3^2)^3}}\{(\overline{B}[V_{\overline{h}_1},V_{\overline{h}_2},V_{\overline{h}_3}])(s_1 + \\ +\overline{\tau}_1,s_2+\overline{\tau}_2,s_3+\overline{\tau}_3;s)\})d\overline{\tau}_1d\overline{\tau}_2d\overline{\tau}_3]\}, \quad (\overline{h}_i = s_i + \overline{\tau}_i;\ i=\overline{1,3}). \end{cases}$$

Here (4.42) – system of the nonlinear integrated equations of Volterra-Abel of the second sort concerning V, W_i on a variable $t \in [0, T_0]$. Therefore, if takes place:

$$\begin{cases} \forall (x_1, x_2, x_3, t) \in T;\ M_1;\ \Pi;\ \overline{\Omega}_i;\ t - s = \tau: \\ \sup\limits_{T} \left| D^k M_1(x_1, x_2, x_3, t) \right| \le \beta_1,\ (k = \overline{0,3}), \\ \sup\limits_{T \times T} \Pi(x_1, x_2, x_3, \tau_1, \tau_2, \tau_3; t, \tau) \equiv \sup\limits_{T \times T} \{ d^{-1} \sum\limits_{i=1}^{3} (\sum\limits_{j=1}^{3} \lambda_j \left| \overline{\Omega}_{il_j}(x_1 + 2\tau_1\sqrt{\mu\tau}, x_2 + 2\tau_2\sqrt{\mu\tau}, x_3 + 2\tau_3 \times \right. \\ \left. \times \sqrt{\mu\tau}; t - \tau) \right|) + \sum\limits_{j=1}^{3} \left| \overline{\Omega}_j(x_1 + 2\tau_1\sqrt{\mu\tau}, x_2 + 2\tau_2\sqrt{\mu\tau}, x_3 + 2\tau_3\sqrt{\mu\tau}; t - \tau) \right| d^{-1} \times \\ \times [\frac{1}{4\pi} \int\limits_{R^3} (\sum\limits_{i=1}^{3} \left| \overline{\tau}_i \right| \frac{1}{\sqrt{(\overline{\tau}_1^2 + \overline{\tau}_2^2 + \overline{\tau}_3^2)^3}} \{ \sum\limits_{i=1}^{3} (\sum\limits_{j=1}^{3} \lambda_j \left| \overline{\Omega}_{il_j}(x_1 + 2\tau_1\sqrt{\mu\tau} + \overline{\tau}_1, x_2 + 2\tau_2\sqrt{\mu\tau} + \overline{\tau}_2, x_3 + \right. \\ \left. + 2\tau_3\sqrt{\mu\tau} + \overline{\tau}_3; t - \tau) \right|) + \sum\limits_{i=1}^{3} (\sum\limits_{j=1}^{3} \left| \overline{\Omega}_{jl_i}(x_1 + 2\tau_1\sqrt{\mu\tau} + \overline{\tau}_1, x_2 + 2\tau_2\sqrt{\mu\tau} + \overline{\tau}_2, x_3 + 2\tau_3\sqrt{\mu\tau} + \right. \\ \left. + \overline{\tau}_3; t - \tau) \right|) \lambda_i \}) d\overline{\tau}_1 d\overline{\tau}_2 d\overline{\tau}_3] \} \le \beta_2 \mu, \\ k_0 = \frac{1}{\sqrt{\pi^3}} \sup\limits_{T} \int\limits_0^t \int\limits_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2)) \Pi(x_1, x_2, x_3, \tau_1, \tau_2, \tau_3; t, \tau) d\tau_1 d\tau_2 d\tau_3 d\tau \le \beta_2 \mu T_0, \\ k_i = \frac{1}{\sqrt{\pi^3}} \sup\limits_{T} \int\limits_0^t \int\limits_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2)) \Pi(x_1, x_2, x_3, \tau_1, \tau_2, \tau_3; t, \tau) \frac{|\tau_i|}{\sqrt{\mu\tau}} d\tau_1 d\tau_2 d\tau_3 d\tau \le \\ \le \sqrt{2T_0}\beta_2\sqrt{\mu}, (i = \overline{1,3}), \quad \beta = \max(\beta_2 T_0; 3\sqrt{2T_0}\beta_2), \end{cases} \tag{4.43}$$

and

$$\begin{cases} \Psi_i, (i = \overline{0,3}):\ k_i \le \frac{h}{4}, (h < 1), \\ \sum\limits_{i=0}^{3} k_i \le \sqrt{\mu}(\beta_2 T_0 \sqrt{\mu} + 3\sqrt{2T_0}\beta_2) \le \sqrt{\mu}(\sqrt{\mu} + 1)\beta = h < 1, \\ S_{r_1}(0) = \{ V, W_i : |V|;\ |W_i| \le r_1, \forall (x_1, x_2, x_3, t) \in T \}, (i = \overline{1,3}), \quad \|\Psi_i[0,0,0,0]\|_C \le r_1(1-h): \\ \|\Psi_i[V, W_1, W_2, W_3]\|_C \le \|\Psi_i[V, W_1, W_2, W_3] - \Psi_i[0,0,0,0]\|_C + \|\Psi_i[0,0,0,0]\|_C \le \\ \le k_i 4 r_1 + r_1(1-h) \le h r_1 + r_1(1-h) = r_1, \\ \Psi_i : S_{r_1}(0) \to S_{r_1}(0),\ (i = \overline{0,3}). \end{cases} \tag{4.44}$$

Then on the basis of (4.31) - (4.34) there is a single solution of system (4.42), and this decision is found on the basis of a Picard's method. Therefore it means that sequence $\{\upsilon_{i,n}\}_0^\infty$ converging to a limit $\upsilon_i, (i = \overline{1,3}):$

$$\upsilon_{i,n+1} \xrightarrow[n \to \infty]{h<1} \upsilon_i, \forall (x_1, x_2, x_3, t) \in T, (i = \overline{1,3}). \tag{4.45}$$

Theorem 5. Under conditions (1.2), (1.3), (A_3), (4.36)-(4.38) and (4.45) problem Navier-Stokes has the single solution in $G^1_{n=3}(D_0)$ in a kind (4.37).

Remarks: I. From the received results follows that a problem Navier-Stokes (1.1)-(1.3) in a case (A_3), (4.36) and (4.45), with smooth enough initial data has the conditional-smooth and single solution in $G^1_{n=3}(D_0)$, so as $V \in G^1(D_0)$.

II. Let's note, if the condition (4.47) is not carried out, but and in this case the system (4.45) has the continuous single solution. As the system (4.45) consists of the integrated equations of Volterra-Abel on a variable $t\in[0, T_0]$.

III. Updating of a Method (4.2), when $\mathrm{div} f \neq 0$. The algorithm (4.37) also is applicable in a case, if

$$\begin{cases} f_i \equiv \varphi_i + \overline{\Omega}_{it},\ (i=\overline{1,3}),\ \mathrm{div} f \neq 0: \quad \mathrm{div}\varphi \neq 0;\ \sum_{i=1}^{3} \frac{\partial}{\partial x_i}\overline{\Omega}_{it} = 0, \\ \mathrm{div}\overline{f} = 0;\ \ \mathrm{rot}\,\overline{f} = 0: \quad \Delta \overline{f_i} = 0, (\Delta\overline{\Omega}_i = 0; i = \overline{1,3}), \\ \upsilon_i\big|_{t=0} = \upsilon_{i0}(x_1,x_2,x_3) \equiv \lambda_i \vartheta_0(x_1,x_2,x_3),\ (\sum_{i=1}^{3}\lambda_i \vartheta_{0x_i} = 0), \end{cases} \tag{4.46}$$

that on a basis (4.39)-(4.41):

$$\begin{cases} \sum_{i=1}^{3}\frac{\partial}{\partial x_i}(4.39):\quad \Delta(\frac{1}{\rho}P + \frac{1}{2}\overline{U}) = -\{\overline{B}[V_{x_1},V_{x_2},V_{x_3}] - \mathrm{div}\varphi\}, \\ (\overline{B}[V_{x_1},V_{x_2},V_{x_3}])(x_1,x_2,x_3,t) \equiv \sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j\overline{\Omega}_{ix_j})V_{x_i} + \sum_{i=1}^{3}(\sum_{j=1}^{3}\overline{\Omega}_{jx_i}V_{x_j})\lambda_i, \\ \frac{1}{\rho}P + \frac{1}{2}\overline{U} = \frac{1}{4\pi}\int_{R^3}\frac{1}{r}\{(\overline{B}[V_{s_1},V_{s_2},V_{s_3}])(s_1,s_2,s_3,t) - \sum_{i=1}^{3}\varphi_{is_i}(s_1,s_2,s_3,t)\}ds_1ds_2ds_3, \\ \frac{1}{\rho}P_{x_i} + \frac{1}{2}\overline{U}_{x_i} = \frac{1}{4\pi}\int_{R^3}\frac{\tau_i}{\sqrt{(\tau_1^2+\tau_2^2+\tau_3^2)^3}}\{(\overline{B}[V_{h_1},V_{h_2},V_{h_3}])(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t) - \\ -\sum_{i=1}^{3}\varphi_{ih_i}(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t)\}d\tau_1d\tau_2d\tau_3,\quad (s_i - x_i = \tau_i; h_i = x_i+\tau_i; i=\overline{1,3}), \\ V_t + d^{-1}V[\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j\overline{\Omega}_{ix_j})] + \sum_{j=1}^{3}V_{x_j}\overline{\Omega}_j = \Phi_0 + (Q[V,V_{x_1},V_{x_2},V_{x_3}])(x_1,x_2,x_3,t) + \mu\Delta V, (i=\overline{1,3}), \\ (Q[V,V_{x_1},V_{x_2},V_{x_3}])(x_1,x_2,x_3,t) \equiv -d^{-1}[\frac{1}{4\pi}\int_{R^3}(\sum_{i=1}^{3}\frac{\tau_i}{\sqrt{(\tau_1^2+\tau_2^2+\tau_3^2)^3}}(\overline{B}[V_{h_1},V_{h_2},V_{h_3}])(x_1 + \\ +\tau_1,x_2+\tau_2,x_3+\tau_3;t))d\tau_1d\tau_2d\tau_3], \\ U = \sum_{j=1}^{3}\overline{\Omega}_j^2;\quad \sum_{j=1}^{3}\overline{\Omega}_j\overline{\Omega}_{ix_j} = \frac{1}{2}\overline{U}_{x_i}, (i=\overline{1,3}),\quad d = \sum_{i=1}^{3}\lambda_i > 0;\ \mathrm{div}\varphi \neq 0, \\ \Phi_0 \equiv d^{-1}[\sum_{i=1}^{3}\varphi_i + \frac{1}{4\pi}\int_{R^3}\{\sum_{i=1}^{3}\frac{\tau_i}{\sqrt{(\tau_1^2+\tau_2^2+\tau_3^2)^3}}(\sum_{i=1}^{3}\varphi_{ih_i}(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t))\}d\tau_1d\tau_2d\tau_3], \end{cases}$$

we will receive (4.42):

$$\begin{cases} V = \Psi_0[V, W_1, W_2, W_3], \\ W_i = \Psi_i[V, W_1, W_2, W_3], (i = \overline{1,3}), \\ V_{x_i} = W_i, \forall (x_1, x_2, x_3, t) \in T, (i = \overline{1,3}). \end{cases} \quad (4.42)^*$$

Hence, as takes place (4.47), that the solution of system (4.42)* we can find on the basis of Picard's method. Then on a basis (4.31)-(4.34) we have (4.45).

Further, we will receive similar results in the conditions of the theorem 5, i.e. under conditions (1.2), (1.3), (A_3), (4.45), (4.46) problem Navier-Stokes has the single solution in $G^1_{n=3}(D_0)$ in a kind (4.37).

5. Problem Navier - Stokes with of Viscosity, when (A_3), $1 < \mu_0 = \mu < \infty$

In problems Navier-Stokes with great values of viscosity components the speed can be so small that in many researches do not consider the forces of inertia.

Let's consider a fluid with viscosity with Reynolds small number where all inertial participants contain in equations Navier-Stokes. As has been said, theoretically, it is not investigated [12]. Hence, here we will consider, methods of integration of the equations Navier-Stokes. However, it is necessary to notice that technical applications of the theory of a liquid with very great values of viscosity, short of the theory of greasing [12], it is rather limited. But theoretically researches in this direction from the scientific point of view are very important. Therefore in this paragraph methods are developed for the decision of these problems in space $\tilde{C}^{3,1}_{n=3}(T)$. Alternatively, we can consider, e.g., a class of suitable solutions constructed in $G^1_{n=3}(D_0)$ on the basis of lemma K. Friedrichs [15].

5.1. Fluid with Average Viscosity by Conditions (A_3), $\mathrm{div} f = 0$

Therefore, in this case, if the initial data $(\upsilon_{i0}; f_i)$ is set in a kind:

$$\begin{cases} \upsilon_i\big|_{t=0} = \upsilon_{i0}(x_1, x_2, x_3) \equiv \lambda_i \vartheta_0(x_1, x_2, x_3), \forall (x_1, x_2, x_3) \in R^3, (i = \overline{1,3}), \\ \sum_{i=1}^{3} \lambda_i \vartheta_{0x_i} = 0, \ (0 < \lambda_i = \mathrm{const}; 1 < \mu_0 = \mu < \infty), \\ f_i \equiv \varphi_i + K_{it}, \ [K_{it} \equiv \frac{1}{\sqrt{\mu}} \bar{f}_i; \ i = \overline{1,3}], \\ \mathrm{div} f = 0 : \ \mathrm{div}\varphi = 0; \ \sum_{i=1}^{3} \frac{\partial}{\partial x_i} K_{it} = 0, \\ \mathrm{div}\bar{f} = 0; \ \mathrm{rot}\,\bar{f} = 0 : \ \Delta \bar{f}_i = 0, (\Delta K_i = 0; i = \overline{1,3}), \\ U = \sum_{j=1}^{3} K_j^2; \ \sum_{j=1}^{3} K_j K_{ix_j} = \frac{1}{2}(\sum_{j=1}^{3} K_j^2)_{x_i} = \frac{1}{2} U_{x_i}, (i = \overline{1,3}), \\ K_i \equiv \frac{1}{\sqrt{\mu}} \int_0^t \bar{f}_i(x_1, x_2, x_3, s) ds; \ \ \varphi = (\varphi_1, \varphi_2, \varphi_3); \ \ \bar{f} = (\bar{f}_1, \bar{f}_2, \bar{f}_3), \end{cases} \quad (5.1)$$

we enter for definition a component of speeds:

$$\begin{cases} \upsilon_i = \lambda_i V(x_1,x_2,x_3,t) + K_i(x_1,x_2,x_3,t), (i=\overline{1,3}), \\ V\big|_{t=0} = \vartheta_0(x_1,x_2,x_3), \quad \forall (x_1,x_2,x_3) \in R^3, \\ \operatorname{div} \nu = 0: \\ \sum_{i=1}^{3} \lambda_i V_{x_i} = 0; \quad \sum_{i=1}^{3} K_{ix_i} = 0, \end{cases} \tag{5.2}$$

at that

$$\begin{cases} \sum_{j=1}^{3} \upsilon_j \upsilon_{ix_j} \equiv V \sum_{j=1}^{3} \lambda_j K_{ix_j} + \lambda_i \sum_{j=1}^{3} V_{x_j} K_j + \frac{1}{2} U_{x_i}, (\lambda_i V \sum_{j=1}^{3} \lambda_j V_{x_j} = 0), \\ \upsilon_{it} \equiv \lambda_i V_t + K_{it}; \quad \mu \Delta \upsilon_i \equiv \mu\{\lambda_i \Delta V + \Delta K_i\} = \mu \lambda_i \Delta V, (i=\overline{1,3}). \end{cases} \tag{5.3}$$

Then on the basis (5.2), (5.3) we will receive

$$\lambda_i V_t + V \sum_{j=1}^{3} \lambda_j K_{ix_j} + \lambda_i \sum_{j=1}^{3} V_{x_j} K_j + \frac{1}{2} U_{x_i} = \varphi_i - \frac{1}{\rho} P_{x_i} + \mu \lambda_i \Delta V, (i=\overline{1,3}). \tag{5.4}$$

Hence on a basis (5.4), we have

$$\begin{cases} \Delta(\frac{1}{\rho} P + \frac{1}{2} U) = -B_*[V_{x_1}, V_{x_2}, V_{x_3}], \\ (B_*[V_{x_1}, V_{x_2}, V_{x_3}])(x_1,x_2,x_3,t) \equiv \sum_{i=1}^{3} (\sum_{j=1}^{3} \lambda_j K_{ix_j}) V_{x_i} + \sum_{i=1}^{3} (\sum_{j=1}^{3} K_{jx_i} V_{x_j}) \lambda_i, \\ \frac{1}{\rho} P + \frac{1}{2} U = \frac{1}{4\pi} \int_{R^3} \frac{1}{r} \{(B_*[V_{s_1}, V_{s_2}, V_{s_3}])(s_1,s_2,s_3,t)\} ds_1 ds_2 ds_3, \\ \frac{1}{\rho} P_{x_i} + \frac{1}{2} U_{x_i} = \frac{1}{4\pi} \int_{R^3} \frac{\tau_i}{\sqrt{(\tau_1^2 + \tau_2^2 + \tau_3^2)^3}} \{(B_*[V_{h_1}, V_{h_2}, V_{h_3}])(x_1+\tau_1, x_2+\tau_2, x_3+ \\ +\tau_3; t)\} d\tau_1 d\tau_2 d\tau_3, \quad (s_i - x_i = \tau_i; h_i = x_i + \tau_i; i = \overline{1,3}), \end{cases} \tag{5.5}$$

so as

$$\begin{cases} \sum_{i=1}^{3} \frac{\partial}{\partial x_i}(5.4): \quad \frac{\partial}{\partial t}[\sum_{i=1}^{3} \lambda_i V_{x_i}(x_1,x_2,x_3,t)] = 0, \\ \sum_{i=1}^{3} \frac{\partial}{\partial x_i} \frac{1}{2} U_{x_i} \equiv \Delta U; \quad \sum_{i=1}^{3} \frac{\partial}{\partial x_i}(-\frac{1}{\rho} P_{x_i}) \equiv -\frac{1}{\rho} \Delta P; \quad \mu \sum_{i=1}^{3} \frac{\partial}{\partial x_i}(\lambda_i \Delta V) = 0, \\ \sum_{i=1}^{3} \frac{\partial}{\partial x_i}(V \sum_{j=1}^{3} \lambda_j K_{ix_j}) \equiv \sum_{i=1}^{3} \lambda_i (\sum_{j=1}^{3} V_{x_j} K_{jx_i}) + \sum_{j=1}^{3} K_j \frac{\partial}{\partial x_j}(\sum_{i=1}^{3} \lambda_i V_{x_i})_{x_j} = \sum_{i=1}^{3} \lambda_i (\sum_{j=1}^{3} V_{x_j} K_{jx_i}), \\ \sum_{i=1}^{3} \frac{\partial}{\partial x_i}(\lambda_i \sum_{j=1}^{3} V_{x_j} K_j) \equiv \sum_{i=1}^{3} V_{x_i} (\sum_{j=1}^{3} \lambda_j K_{ix_j}) + V \sum_{j=1}^{3} \lambda_j (\sum_{i=1}^{3} K_{ix_i})_{x_j} = \sum_{i=1}^{3} V_{x_i} (\sum_{j=1}^{3} \lambda_j K_{ix_j}), \\ \sum_{j=1}^{3} K_j \frac{\partial}{\partial x_j}(\sum_{i=1}^{3} \lambda_i V_{x_i})_{x_j} = 0; \quad V \sum_{j=1}^{3} \lambda_j (\sum_{i=1}^{3} K_{ix_i})_{x_j} = 0; \quad \operatorname{div} \varphi = 0. \end{cases}$$

Then system (5.4) it is equivalent will be transformed to a kind

$$\begin{cases} V_t + d^{-1}V[\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j K_{ix_j})] + \sum_{j=1}^{3} V_{x_j} K_j = \Phi_0 - d^{-1}[\frac{1}{4\pi}\int_{R^3}(\sum_{i=1}^{3}\frac{\tau_i}{\sqrt{(\tau_1^2+\tau_2^2+\tau_3^2)^3}}\times \\ \times\{(B_*[V_{h_1},V_{h_2},V_{h_3}])(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t)\})d\tau_1 d\tau_2 d\tau_3] + \mu\Delta V,(h_i = x_i+\tau_i; i=\overline{1,3}), \\ \Phi_0 \equiv d^{-1}\sum_{i=1}^{3}\varphi_i; \quad d = \sum_{i=1}^{3}\lambda_i > 0. \end{cases} \tag{5.6}$$

For consideration of unknown function V we have

$$\begin{cases} V = \frac{1}{8\sqrt{(\mu\pi t)^3}}\int_{R^3}\exp(-\frac{r^2}{4\mu t})\vartheta_0(s_1,s_2,s_3)ds_1 ds_2 ds_3 + \frac{1}{8\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-\frac{r^2}{4\mu(t-s)})\times \\ \times(Q[V,V_{s_1},V_{s_2},V_{s_3}])(s_1,s_2,s_3,s)\frac{1}{(\sqrt{\mu(t-s)})^3}ds_1 ds_2 ds_3 ds, \\ M_1(x_1,x_2,x_3,t) \equiv \frac{1}{\sqrt{\pi^3}}\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\vartheta_0(x_1+2\tau_1\sqrt{\mu t},x_2+2\tau_2\sqrt{\mu t},x_3+2\tau_3\times \\ \times\sqrt{\mu t})d\tau_1 d\tau_2 d\tau_3\frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Phi_0(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)}, \\ x_3+2\tau_3\sqrt{\mu(t-s)};s)d\tau_1 d\tau_2 d\tau_3 ds, \\ s_j - x_j = 2\tau_j\sqrt{\mu t};\ s_j - x_j = 2\tau_j\sqrt{\mu(t-s)},(j=\overline{1,3}), \\ (Q[V,V_{s_1},V_{s_2},V_{s_3}])(s_1,s_2,s_3,s) \equiv -\{d^{-1}[V(s_1,s_2,s_3,s)\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j K_{is_j}(s_1,s_2,s_3,s))] + \\ +\sum_{j=1}^{3}V_{s_j}(s_1,s_2,s_3,s)\times K_j(s_1,s_2,s_3,s) + d^{-1}[\frac{1}{4\pi}\int_{R^3}(\sum_{i=1}^{3}\bar\tau_i\frac{1}{\sqrt{(\bar\tau_1^2+\bar\tau_2^2+\bar\tau_3^2)^3}}\times \\ \times\{(B_*[V_{\bar h_1},V_{\bar h_2},V_{\bar h_3}])(s_1+\bar\tau_1,s_2+\bar\tau_2,s_3+\bar\tau_3;s)\})d\bar\tau_1 d\bar\tau_2 d\bar\tau_3]\}, \\ \bar h_j = s_j + \bar\tau_j,(j=\overline{1,3}). \end{cases} \tag{5.7}$$

Hence, differentiating (5.7) on x_i and having entered designation:

$$V_{x_i} = W_i, \forall(x_1,x_2,x_3,t)\in T,(i=\overline{1,3}), \tag{5.8}$$

from (5.7) we will receive

$$\begin{cases} V = M_1(x_1,x_2,x_3,t) + \frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))(Q[V,W_1,W_2,W_3])(x_1+2\tau_1\sqrt{\mu(t-s)}, \\ x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)d\tau_1 d\tau_2 d\tau_3 ds \equiv \Psi_0[V,W_1,W_2,W_3], \\ W_i = M_{1x_i} + \frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\frac{\tau_i}{\sqrt{\mu(t-s)}}(Q[V,W_1,W_2,W_3])(x_1+2\tau_1\sqrt{\mu(t-s)}, \\ x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)d\tau_1 d\tau_2 d\tau_3 ds \equiv \Psi_i[V,W_1,W_2,W_3], \quad (i=\overline{1,3}). \end{cases} \tag{5.9}$$

If takes place:

$$
\begin{cases}
\forall(x_1,x_2,x_3,t)\in T;M_1;Y;K_i,(t-s=\tau):\ \sup\limits_T\left|D^kM_1(x_1,x_2,x_3,t)\right|\le\beta_1,\ (k=\overline{0,3}),\\
\sup\limits_{T\times T}Y(x_1,x_2,x_3,\tau_1,\tau_2,\tau_3;t,\tau)\equiv\sup\limits_{T\times T}\{d^{-1}\sum\limits_{i=1}^{3}(\sum\limits_{j=1}^{3}\lambda_j\left|K_{is_j}(x_1+2\tau_1\sqrt{\mu\tau},x_2+\right.\\
\left.+2\tau_2\sqrt{\mu\tau},x_3+2\tau_3\sqrt{\mu\tau};t-\tau)\right|)+\sum\limits_{j=1}^{3}\left|K_j(x_1+2\tau_1\sqrt{\mu\tau},x_2+2\tau_2\sqrt{\mu\tau},x_3+\right.\\
\left.+2\tau_3\sqrt{\mu\tau};t-\tau)\right|+d^{-1}[\frac{1}{4\pi}\int\limits_{R^3}(\sum\limits_{i=1}^{3}\left|\overline{\tau}_i\right|\frac{1}{\sqrt{(\overline{\tau}_1^{2}+\overline{\tau}_2^{2}+\overline{\tau}_3^{2})^3}}\{\sum\limits_{i=1}^{3}(\sum\limits_{j=1}^{3}\lambda_j\left|K_{il_j}(x_1+\right.\\
\left.+2\tau_1\sqrt{\mu\tau}+\overline{\tau}_1,x_2+2\tau_2\sqrt{\mu\tau}+\overline{\tau}_2,x_3+2\tau_3\sqrt{\mu\tau}+\overline{\tau}_3;t-\tau)\right|)+\sum\limits_{i=1}^{3}(\sum\limits_{j=1}^{3}\left|K_{jl_i}(x_1+\right.\\
\left.+2\tau_1\sqrt{\mu\tau}+\overline{\tau}_1,x_2+2\tau_2\sqrt{\mu\tau}+\overline{\tau}_2,x_3+2\tau_3\sqrt{\mu\tau}+\overline{\tau}_3;t-\tau)\right|)\lambda_i\})d\overline{\tau}_1d\overline{\tau}_2d\overline{\tau}_3]\}\le\frac{1}{\sqrt{\mu}}\beta_2,\\
k_0=\frac{1}{\sqrt{\pi^3}}\sup\limits_T\int\limits_0^t\int\limits_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))Y(x_1,x_2,x_3,\tau_1,\tau_2,\tau_3;t,\tau)d\tau_1d\tau_2d\tau_3d\tau\le\frac{1}{\sqrt{\mu}}\beta_2T_0,\\
k_i=\frac{1}{\sqrt{\pi^3}}\sup\limits_T\int\limits_0^t\int\limits_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))Y(x_1,x_2,x_3,\tau_1,\tau_2,\tau_3;t,\tau)\frac{\left|\tau_i\right|}{\sqrt{\mu\tau}}\times\\
\times d\tau_1d\tau_2d\tau_3d\tau\le\sqrt{2T_0}\beta_2\frac{1}{\mu},(i=\overline{1,3}),\quad \beta=\max(\beta_2T_0;\ 3\sqrt{2T_0}\beta_2),
\end{cases}
\tag{5.10}
$$

and

$$
\begin{cases}
\Psi_i,(i=\overline{0,3}):\quad k_i\le\frac{h}{4},(h<1),\\
\sum\limits_{i=0}^{3}k_i\le\frac{1}{\sqrt{\mu}}(\beta_2T_0+3\sqrt{2T_0}\beta_2\frac{1}{\sqrt{\mu}})\le\frac{1}{\sqrt{\mu}}(1+\frac{1}{\sqrt{\mu}})\beta=h<1,\quad(1<\mu=\mu_0<\infty),\\
S_{r_1}(0)=\{V,W_i:\left|V\right|;\ \left|W_i\right|\le r_1,\forall(x_1,x_2,x_3,t)\in T\},(i=\overline{1,3}),\quad \left\|\Psi_i[0,0,0,0]\right\|_C\le r_1(1-h):\\
\left\|\Psi_i[V,W_1,W_2,W_3]\right\|_C\le\left\|\Psi_i[V,W_1,W_2,W_3]-\Psi_i[0,0,0,0]\right\|_C+\left\|\Psi_i[0,0,0,0]\right\|_C\le\\
\le k_i4r_1+r_1(1-h)\le hr_1+r_1(1-h)=r_1,\\
\Psi_i:S_{r_1}(0)\to S_{r_1}(0),\ (i=\overline{0,3}).
\end{cases}
\tag{5.11}
$$

Then the solution (5.9) exists and is unique, and we can find this decision on the basis of a Picard's method

$$
\begin{cases}
\forall(x_1,x_2,x_3,t)\in T:\quad V_{n+1}=\Psi_0[V_n,W_{1,n},W_{2,n},W_{3,n}],\\
W_{i,n+1}=\Psi_i[V_n,W_{1,n},W_{2,n},W_{3,n}],(n=0,1,\ldots;\ V_0=0;\ W_{i,0}=0;\ i=\overline{1,3}).
\end{cases}
\tag{5.12}
$$

Therefore using known mathematical conclusions concerning of Picard's method we can tell the following. Received the sequence of functions $\{V_n\}_0^\infty, \{W_{i,n}\}_0^\infty, (i=\overline{1,3})$ is converging and fundamental in $S_{r_1}(0)$, i.e.:

$$\begin{cases} E_{n+1} = \|V_{n+1} - V_n\|_C + \sum_{i=1}^{3} \|W_{i,n+1} - W_{i,n}\|_C ;\; E_n = \|V_n - V_{n-1}\|_C + \sum_{i=1}^{3} \|W_{i,n} - W_{i,n-1}\|_C : \\ \|V_{n+1} - V_n\|_C \le k_0 E_n ;\quad \|W_{i,n+1} - W_{i,n}\|_C \le k_i E_n ,(i=\overline{1,3}), \\ E_{n+1} \le hE_n \le \ldots \le h^n E_1 \xrightarrow[n\to\infty]{h<1} 0, \\ \|V_{n+k} - V_n\|_C \le \sum_{j=0}^{k-1} k_0 E_{n+j} ;\quad \|W_{i,n+k} - W_{i,n}\|_C \le \sum_{j=0}^{k-1} k_i E_{n+j} ,(i=\overline{1,3}), \\ E_{n+k} \le h\sum_{j=0}^{k-1} E_{n+j} \le \ldots \le h\sum_{j=0}^{k-1} h^{n+j-1}E_1 \le E_1 h^n \sum_{j=0}^{k-1} h^j \le E_1 h^n \dfrac{1}{1-h} \xrightarrow[n\to\infty]{h<1} 0, \\ X_0 = \|V\|_C + \sum_{i=1}^{3} \|W_i\|_C ;\; X_{n+1} \equiv \|V_{n+1} - V\|_C + \sum_{i=1}^{3} \|W_{i,n+1} - W_i\|_C : \\ X_{n+1} \le h^{n+1} X_0 \xrightarrow[n\to\infty]{h<1} 0, \\ V_{n+1} \xrightarrow[n\to\infty]{h<1} V \equiv H ;\quad W_{i,n+1} \xrightarrow[n\to\infty]{h<1} W_i ,\forall (x_1,x_2,x_3,t) \in T ,(i=\overline{1,3}). \end{cases} \tag{5.13}$$

Therefore on the basis of (5.2) we will receive

$$\begin{cases} \upsilon_{i,n+1} = \lambda_i[\vartheta_0(x_1,x_2,x_3) + V_{n+1}(x_1,x_2,x_3,t)] + K_i(x_1,x_2,x_3,t),(n=0,1,2,\ldots; i=\overline{1,3}), \\ \|\upsilon_{i,n+1} - \upsilon_i\|_C \le \lambda_i \|V_{n+1} - V\|_C \le \lambda_i h X_n \le \lambda_i h^{n+1} X_0 \xrightarrow[n\to\infty]{h<1} 0,(i=\overline{1,3}), \end{cases} \tag{5.14}$$

i.e. it means that sequence $\{\upsilon_{i,n}\}_0^\infty$ converging to a limit $\upsilon_i,(i=\overline{1,3}):$

$$\upsilon_{i,n+1} \xrightarrow[n\to\infty]{h<1} \upsilon_i ,\forall (x_1,x_2,x_3,t) \in T ,(i=\overline{1,3}). \tag{5.15}$$

Theorem 6. Under conditions (1.2), (1.3), (A_3), (5.1), (5.15) problem Navier-Stokes has the continuous single solution, which it is found by a rule (5.2) in $G^1_{n=3}(D_0)$.

Remarks:

I. Singleness is obvious, as a method by contradiction. Results (5.13) with a condition ((A_3), (5.1), (5.11), (5.12)) are received where smoothness of functions is required only on x_i as the derivative of 1st order is in time has $t>0$. Then taking into account (5.13) the system (5.9) has the continuous single solution $V \in C^{3,0}(T)$. Therefore, $V \in G^1(D_0)$. Hence, on the basics (5.2), (5.14), (5.15) $\upsilon_i \in G^1(D_0)$.

II. The algorithm (5.2) also is applicable in a case, if

$$\begin{cases} f_i \equiv \varphi_i + K_{it},\ \mathrm{div} f \ne 0 :\ \mathrm{div}\varphi \ne 0;\ \sum_{i=1}^{3} \dfrac{\partial}{\partial x_i} K_{it} = 0, \\ \mathrm{div}\overline{f} = 0;\ \mathrm{rot}\,\overline{f} = 0 :\ \Delta \overline{f_i} = 0,(\Delta K_i = 0; i=\overline{1,3}), \\ U = \sum_{j=1}^{3} K_j^2 ;\quad \sum_{j=1}^{3} K_j K_{ix_j} = \dfrac{1}{2} U_{x_i}, \\ \upsilon_i\big|_{t=0} = \upsilon_{i0}(x_1,x_2,x_3) \equiv \lambda_i \vartheta_0(x_1,x_2,x_3),(i=\overline{1,3}). \end{cases} \tag{5.16}$$

That on a basis (5.16), (5.2) we will receive (5.4). Hence, we have

$$\begin{cases} \operatorname{div}\varphi \neq 0; \quad \sum_{i=1}^{3} \frac{\partial}{\partial x_i}(5.4): \quad \Delta\left(\frac{1}{\rho}P + \frac{1}{2}U\right) = -\{B_*[V_{x_1}, V_{x_2}, V_{x_3}] - \operatorname{div}\varphi\}, \\ (B_*[V_{x_1}, V_{x_2}, V_{x_3}])(x_1, x_2, x_3, t) \equiv \sum_{i=1}^{3}\left(\sum_{j=1}^{3} \lambda_j K_{ix_j}\right)V_{x_i} + \sum_{i=1}^{3}\left(\sum_{j=1}^{3} K_{jx_i} V_{x_j}\right)\lambda_i, \\ \frac{1}{\rho}P + \frac{1}{2}U = \frac{1}{4\pi}\int_{R^3} \frac{1}{r}\{(B_*[V_{s_1}, V_{s_2}, V_{s_3}])(s_1, s_2, s_3, t) - \sum_{i=1}^{3} \varphi_{is_i}(s_1, s_2, s_3, t)\} ds_1 ds_2 ds_3, \\ \frac{1}{\rho}P_{x_i} + \frac{1}{2}U_{x_i} = \frac{1}{4\pi}\int_{R^3} \frac{\tau_i}{\sqrt{(\tau_1^2 + \tau_2^2 + \tau_3^2)^3}}\{(B_*[V_{h_1}, V_{h_2}, V_{h_3}])(x_1 + \tau_1, x_2 + \tau_2, x_3 + \tau_3; t) - \\ -\sum_{i=1}^{3} \varphi_{ih_i}(x_1 + \tau_1, x_2 + \tau_2, x_3 + \tau_3; t)\} d\tau_1 d\tau_2 d\tau_3, \quad (s_i - x_i = \tau_i; h_i = x_i + \tau_i), \end{cases} \tag{5.17}$$

and

$$\begin{cases} V_t + d^{-1}V[\sum_{i=1}^{3}(\sum_{j=1}^{3} \lambda_j K_{ix_j})] + \sum_{j=1}^{3} V_{x_j} K_j = \Phi_0 - d^{-1}[\frac{1}{4\pi}\int_{R^3} (\sum_{i=1}^{3} \frac{\tau_i}{\sqrt{(\tau_1^2 + \tau_2^2 + \tau_3^2)^3}} \times \\ \times\{(B_*[V_{h_1}, V_{h_2}, V_{h_3}])(x_1 + \tau_1, x_2 + \tau_2, x_3 + \tau_3; t)\}) d\tau_1 d\tau_2 d\tau_3] + \mu\Delta V, (i = \overline{1,3}), \\ \Phi_0(x_1, x_2, x_3, t) \equiv d^{-1}[\sum_{i=1}^{3} \varphi_i + \frac{1}{4\pi}\int_{R^3} \{\sum_{i=1}^{3} \tau_i \frac{1}{\sqrt{(\tau_1^2 + \tau_2^2 + \tau_3^2)^3}}(F_0(x_1 + \tau_1, x_2 + \tau_2, x_3 + \\ +\tau_3; t))\} d\tau_1 d\tau_2 d\tau_3 + \mu\sum_{i=1}^{3} \Delta K_i], \quad (d = \sum_{i=1}^{3} \lambda_i > 0; \ F_0(x_1, x_2, x_3, t) \equiv \sum_{i=1}^{3} \varphi_{ix_i}), \end{cases} \tag{5.18}$$

here (5.17) differs from (5.5), as $\operatorname{div}\varphi \neq 0$. Then considering (5.18), (5.7), (5.8) we will receive system (5.9):

$$\begin{cases} V = \overline{\Psi}_0[V, W_1, W_2, W_3], \\ W_i = \overline{\Psi}_i[V, W_1, W_2, W_3], (i = \overline{1,3}). \end{cases} \tag{5.19}$$

Therefore, as takes place (5.10), (5.11), (5.13) in a consequence (5.14), (5.15), i.e.

$$\begin{cases} \left\|\upsilon_{i,n+1} - \upsilon_i\right\|_C \leq \lambda_i \left\|V_{n+1} - V\right\|_C \leq \lambda_i h X_n \leq \lambda_i h^{n+1} X_0 \xrightarrow[n\to\infty]{h<1} 0, (i = \overline{1,3}), \\ \upsilon_{i,n+1} \xrightarrow[n\to\infty]{h<1} \upsilon_i, \forall(x_1, x_2, x_3, t) \in T, (i = \overline{1,3}). \end{cases} \tag{5.20}$$

Further, we will receive similar results in the conditions of the theorem 6.

5.2. Updating of a Method (5.2) in a Case (A3), $\Delta\vartheta_0 = 0; \ \operatorname{div} f = 0; \ \operatorname{rot} f = 0$

The decision method from where follows of equations integration of Navier-Stokes in $\tilde{C}^1_{n=3}(T)$ in a case

$$\begin{cases} \upsilon_i\big|_{t=0} = \upsilon_{i0}(x_1, x_2, x_3) \equiv \lambda_i\vartheta_0(x_1, x_2, x_3), (0 < \lambda_i = \text{const}; i = \overline{1,3}), \\ \sum_{i=1}^{3} \lambda_i \vartheta_{0x_i} = 0, \ (i = \overline{1,3}), \ \Delta\vartheta_0 = 0; \ \vartheta_0 \in R^3, \end{cases} \tag{5.21}$$

$$\begin{cases} \operatorname{div} f = 0;\ \operatorname{rot} f = 0,\ (\Delta f_i = 0;\quad f = (f_1, f_2, f_3)), \sup\limits_T \left|D^k f_i\right| \le N_0,\ \forall (x_1, x_2, x_3, t) \in T, \\ K_i(x_1, x_2, x_3, t) \equiv \dfrac{1}{\sqrt{\mu}} \int\limits_0^t f_i(x_1, x_2, x_3, s)ds,\ (\Delta K_i = 0;\ i = \overline{1,3}), \end{cases}$$

is a major factor of this point.

Therefore we enter for definition a component of speeds:

$$\begin{cases} \upsilon_i = \lambda_i[\vartheta_0(x_1, x_2, x_3) + Z(x_1, x_2, x_3, t)] + K_i(x_1, x_2, x_3, t), \forall (x_1, x_2, x_3, t) \in T, (i = \overline{1,3}), \\ Z\big|_{t=0} = 0, \forall (x_1, x_2, x_3) \in R^3, \\ \operatorname{div} \nu = 0: \\ \sum\limits_{i=1}^3 \lambda_i Z_{x_i} = 0;\ \sum\limits_{i=1}^3 \lambda_i \vartheta_{0x_i} = 0;\ \sum\limits_{i=1}^3 K_{ix_i} = 0, \end{cases} \tag{5.22}$$

at that

$$\begin{cases} K_{it} \equiv \dfrac{1}{\sqrt{\mu}} f_i(x_1, x_2, x_3, t);\ U = \sum\limits_{j=1}^3 K_j^2;\ \sum\limits_{j=1}^3 K_j K_{ix_j} = \dfrac{1}{2} U_{x_i},\ (i = \overline{1,3}), \\ \sum\limits_{j=1}^3 \upsilon_j \upsilon_{ix_j} \equiv \sum\limits_{j=1}^3 \lambda_j (\vartheta_0 + Z) K_{ix_j} + \sum\limits_{j=1}^3 \lambda_i (\vartheta_{0x_j} + Z_{x_j}) K_j + \dfrac{1}{2} U_{x_i}, \\ \sum\limits_{j=1}^3 \lambda_j (\vartheta_0 + Z) \lambda_i (\vartheta_{0x_j} + Z_{x_j}) = \lambda_i (\vartheta_0 + Z) \sum\limits_{j=1}^3 \lambda_j (\vartheta_{0x_j} + Z_{x_j}) = 0, \\ \upsilon_{it} \equiv \lambda_i Z_t + K_{it};\quad \mu \Delta \upsilon_i \equiv \mu[\lambda_i(\Delta \vartheta_0 + \Delta Z) + \Delta K_i] = \mu \lambda_i \Delta Z,\ (i = \overline{1,3}). \end{cases} \tag{5.23}$$

Hence on the basis of (5.21)-(5.23) we will receive

$$\lambda_i Z_t + \sum_{j=1}^3 \lambda_j (\vartheta_0 + Z) K_{ix_j} + \sum_{j=1}^3 K_j \lambda_i (\vartheta_{0x_j} + Z_{x_j}) + \frac{1}{2} U_{x_i} = (1 - \frac{1}{\sqrt{\mu}}) f_i - \frac{1}{\rho} P_{x_i} + \mu \lambda_i \Delta Z, (i = \overline{1,3}). \tag{5.24}$$

Then for incompressible currents with a friction the equations of Navier-Stokes (1.1) become simpler as take place (5.21), (5.23). Therefore a problem (1.1)-(1.3) with account [(5.5)-(5.9) here instead of Ω_i we will consider K_i] we will receive

$$\begin{cases} Z_{x_i} = W_i, \forall (x_1, x_2, x_3, t) \in T, (i = \overline{1,3}), \\ Z = M_1 + \dfrac{1}{8\sqrt{\pi^3}} \int\limits_0^t \int\limits_{R^3} \exp(-\dfrac{r^2}{4\mu(t-s)}) (Q[Z, W_1, W_2, W_3])(s_1, s_2, s_3, s) \dfrac{ds_1 ds_2 ds_3 ds}{(\sqrt{\mu(t-s)})^3} = M_1 + \\ + \dfrac{1}{\sqrt{\pi^3}} \int\limits_0^t \int\limits_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2))(Q[Z, W_1, W_2, W_3])(x_1 + 2\tau_1\sqrt{\mu(t-s)}, x_2 + 2\tau_2\sqrt{\mu(t-s)}, x_3 + \\ + 2\tau_3\sqrt{\mu(t-s)}; s) d\tau_1 d\tau_2 d\tau_3 ds \equiv (\Gamma_0[Z, W_1, W_2, W_3])(x_1, x_2, x_3, t), \\ W_i = M_{1x_i} + \dfrac{1}{8\sqrt{\pi^3}} \int\limits_0^t \int\limits_{R^3} (\exp(-\dfrac{r^2}{4\mu(t-s)})) \dfrac{-(x_i - s_i)}{2\mu(t-s)} (Q[Z, W_1, W_2, W_3])(s_1, s_2, s_3, s) \dfrac{ds_1 ds_2 ds_3 ds}{(\sqrt{\mu(t-s)})^3} = \end{cases}$$

$$\begin{cases} = M_{1x_i} + \dfrac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\dfrac{\tau_i}{\sqrt{\mu(t-s)}}(Q[Z,W_1,W_2,W_3])(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+ \\ +2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)d\tau_1 d\tau_2 d\tau_3 ds \equiv (\Gamma_i[Z,W_1,W_2,W_3])(x_1,x_2,x_3,t),(i=\overline{1,3}), \end{cases} \tag{5.25}$$

where

$$\begin{cases} \Delta(\dfrac{1}{\rho}P+\dfrac{1}{2}U) = -\{F_0 + B_*[Z_{x_1},Z_{x_2},Z_{x_3}]\}, \\ (B_*[Z_{x_1},Z_{x_2},Z_{x_3}])(x_1,x_2,x_3,t) \equiv \sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j K_{ix_j})Z_{x_i} + \sum_{i=1}^{3}(\sum_{j=1}^{3}K_{jx_i}Z_{x_j})\lambda_i, \\ F_0(x_1,x_2,x_3,t) \equiv \sum_{i=1}^{3}\lambda_i(\sum_{j=1}^{3}K_{jx_i}\vartheta_{0x_j}) + \sum_{i=1}^{3}\vartheta_{0x_i}(\sum_{j=1}^{3}\lambda_j K_{ix_j}), \\ \dfrac{1}{\rho}P+\dfrac{1}{2}U = \dfrac{1}{4\pi}\int_{R^3}\dfrac{1}{r}\{F_0(s_1,s_2,s_3,t)+(B_*[Z_{s_1},Z_{s_2},Z_{s_3}])(s_1,s_2,s_3,t)\}ds_1 ds_2 ds_3, \\ \dfrac{1}{\rho}P_{x_i}+\dfrac{1}{2}U_{x_i} = \dfrac{1}{4\pi}\int_{R^3}\dfrac{\tau_i}{\sqrt{(\tau_1^2+\tau_2^2+\tau_3^2)^3}}\{F_0(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t)+ \\ +(B_*[Z_{h_1},Z_{h_2},Z_{h_3}])(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t)\}d\tau_1 d\tau_2 d\tau_3,(s_i-x_i=\tau_i;h_i=x_i+\tau_i;\ i=\overline{1,3}), \\ Z_t = \Phi_0 + (Q[Z,Z_{x_1},Z_{x_2},Z_{x_3}])(x_1,x_2,x_3,t) + \mu\Delta Z,(i=\overline{1,3}), \\ M_1(x_1,x_2,x_3,t) \equiv \dfrac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Phi_0(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)}, \\ x_3+2\tau_3\sqrt{\mu(t-s)};s)d\tau_1 d\tau_2 d\tau_3 ds, \\ (s_i-x_i=2\tau_i\sqrt{\mu(t-s)};t-s=\tau;i=\overline{1,3}), \\ (Q[Z,Z_{s_1},Z_{s_2},Z_{s_3}])(s_1,s_2,s_3,s) \equiv -\{d_0^{-1}[Z(s_1,s_2,s_3,s)\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j K_{is_j}(s_1,s_2,s_3,s))]+ \\ +\sum_{j=1}^{3}Z_{s_j}(s_1,s_2,s_3,s)K_j(s_1,s_2,s_3,s)+d_0^{-1}[\dfrac{1}{4\pi}\int_{R^3}(\sum_{i=1}^{3}\dfrac{\overline{\tau}_i}{\sqrt{(\overline{\tau}_1^2+\overline{\tau}_2^2+\overline{\tau}_3^2)^3}}\{(B_*[Z_{\overline{h}_1},Z_{\overline{h}_2},Z_{\overline{h}_3}])(s_1+ \\ +\overline{\tau}_1,s_2+\overline{\tau}_2,s_3+\overline{\tau}_3;s)\})d\overline{\tau}_1 d\overline{\tau}_2 d\overline{\tau}_3]\},\ \overline{h}_I = s_i+\overline{\tau}_i,(i=\overline{1,3}), \\ \operatorname{div} f = 0;\quad d_0 = \sum_{i=1}^{3}\lambda_i > 0, \\ \Phi_0 \equiv -\sum_{j=1}^{3}K_j\vartheta_{0x_j} + d_0^{-1}[\sum_{i=1}^{3}(1-\dfrac{1}{\sqrt{\mu}})f_i - \sum_{i=1}^{3}\vartheta_0(\sum_{j=1}^{3}\lambda_j K_{ix_j}) - \dfrac{1}{4\pi}\int_{R^3}\{\sum_{i=1}^{3}\dfrac{\tau_i}{\sqrt{(\tau_1^2+\tau_2^2+\tau_3^2)^3}}\times \\ \times(F_0(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t))\}d\tau_1 d\tau_2 d\tau_3]. \end{cases} \tag{5.26}$$

If the known functions entering into system (5.25) satisfy conditions

$$\begin{cases} \forall(x_1,x_2,x_3,t)\in T;M_1;Y;K_i,\ (t-s=\tau):\ \sup_T\left|D^k M_1(x_1,x_2,x_3,t)\right| \le \beta_1,\ (k=\overline{0,3}), \\ \sup_{T\times T} Y(x_1,x_2,x_3,\tau_1,\tau_2,\tau_3;t,\tau) \equiv \sup_{T\times T}\{d_0^{-1}\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j\left|K_{is_j}(x_1+2\tau_1\sqrt{\mu\tau},x_2+2\tau_2\sqrt{\mu\tau},x_3+\right. \end{cases}$$

$$\begin{cases} +2\tau_3\sqrt{\mu\tau};t-\tau)\big|)+\sum_{j=1}^{3}\big|K_j(x_1+2\tau_1\sqrt{\mu\tau},x_2+2\tau_2\sqrt{\mu\tau},x_3+2\tau_3\sqrt{\mu\tau};t-\tau)\big|+ \\ +d_0^{-1}[\frac{1}{4\pi}\int_{R^3}(\sum_{i=1}^{3}\frac{|\bar{\tau}_i|}{\sqrt{(\bar{\tau}_1^2+\bar{\tau}_2^2+\bar{\tau}_3^2)^3}}\{\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j\big|K_{il_j}(x_1+2\tau_1\sqrt{\mu\tau}+\bar{\tau}_1,x_2+2\tau_2\sqrt{\mu\tau}+\bar{\tau}_2,x_3+ \\ +2\tau_3\sqrt{\mu\tau}+\bar{\tau}_3;t-\tau)\big|)+\sum_{i=1}^{3}(\sum_{j=1}^{3}\big|K_{jl_i}(x_1+2\tau_1\sqrt{\mu\tau}+\bar{\tau}_1,x_2+2\tau_2\sqrt{\mu\tau}+\bar{\tau}_2,x_3+2\tau_3\sqrt{\mu\tau}+ \\ +\bar{\tau}_3;t-\tau)\big|)\lambda_i\})d\bar{\tau}_1d\bar{\tau}_2d\bar{\tau}_3]\}\le\frac{1}{\sqrt{\mu}}\beta_2, \\ k_0=\frac{1}{\sqrt{\pi^3}}\sup_T\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))Y(x_1,x_2,x_3,\tau_1,\tau_2,\tau_3;t,\tau)d\tau_1d\tau_2d\tau_3d\tau\le\frac{1}{\sqrt{\mu}}\beta_2T_0, \\ k_i=\frac{1}{\sqrt{\pi^3}}\sup_T\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))Y(x_1,x_2,x_3,\tau_1,\tau_2,\tau_3;t,\tau)\frac{|\tau_i|}{\sqrt{\mu\tau}}d\tau_1d\tau_2d\tau_3d\tau\le \\ \le\sqrt{2T_0}\beta_2\frac{1}{\mu},(i=\overline{1,3});\quad\beta_*=\max(\beta_2T_0;3\sqrt{2T_0}\beta_2), \end{cases} \tag{5.27}$$

and

$$\begin{cases} \Gamma_i,(i=\overline{0,3}):\ k_i\le\frac{h}{4},(h<1),(i=\overline{1,3}), \\ \sum_{i=0}^{3}k_i\le\frac{1}{\sqrt{\mu}}(\beta_2T_0+3\sqrt{2T_0}\beta_2\frac{1}{\sqrt{\mu}})\le\frac{1}{\sqrt{\mu}}(1+\frac{1}{\sqrt{\mu}})\beta_*=h<1,(1<\mu=\mu_0<\infty), \\ S_{r_1}(0)=\{Z,W_i:|Z|;\ |W_i|\le r_1,\forall(x_1,x_2,x_3,t)\in T\},(i=\overline{1,3}),\quad\|\Gamma_i[0,0,0,0]\|_C\le r_1(1-h): \\ \|\Gamma_i[Z,W_1,W_2,W_3]\|_C\le\|\Gamma_i[Z,W_1,W_2,W_3]-\Gamma_i[0,0,0,0]\|_C+\|\Gamma_i[0,0,0,0]\|_C\le k_i4r_1+r_1(1-h)\le \\ \le hr_1+r_1(1-h)=r_1, \\ \Gamma_i:S_{r_1}(0)\to S_{r_1}(0),\ (i=\overline{0,3}). \end{cases} \tag{5.28}$$

That the solution of this system we can find on the basis of Picard's method

$$\begin{cases} \forall(x_1,x_2,x_3,t)\in T:\quad Z_{n+1}=\Gamma_0[Z_n,W_{1,n},W_{2,n},W_{3,n}], \\ W_{i,n+1}=\Gamma_i[Z_n,W_{1,n},W_{2,n},W_{3,n}],(Z_0=0;\ \ W_{i,0}=0;\ i=\overline{1,3};\ \ n=0,1,\ldots). \end{cases} \tag{5.29}$$

Then considering results (5.13)-(5.15) we will receive, i.e. that sequence $\{\upsilon_{i,n}\}_0^\infty$ converging to a limit

$\upsilon_i,(i=\overline{1,3}):$

$$\upsilon_{i,n+1}\xrightarrow[n\to\infty]{h<1}\upsilon_i,\forall(x_1,x_2,x_3,t)\in T,(i=\overline{1,3}). \tag{5.30}$$

Theorem 6*. If the conditions (1.2), (1.3), (5.21)-(5.23) and (5.30) are fulfilled, then the problem Navier-Stokes has the smooth single solution in $\tilde{C}^{3,1}_{n=3}(T)$.

Remark 3. If takes place $0<\beta_*\le 2^{-1}$, that $1<\mu=\mu_0<\infty$. In a case $\beta_*>2^{-1}$, then

$$m_0 < \mu = \mu_0 = \text{const} < \infty, \ (m_0 > \max[1; 4(\sqrt{1+4\beta_*^{-1}} - 1)^{-2}]). \qquad (1)^*$$

In a case $\mu = 1$, that

$$\sum_{i=0}^{3} k_i \le (\sqrt{T_0} + 3\sqrt{2})\beta_2\sqrt{T_0} \le 2\beta_* = h < 1, \ \text{(see. (5.28))}. \qquad (2)^*$$

Let's note, as the equations of system ($\mu = 1$, (5.25)) are the equations of Volterra-Abel [13] on a variable t, discussing in language of the Volterra's equations, we can find the decision in $\tilde{C}^{3,1}(T)$, when the condition (5.28) is not satisfied. For simplicity, let's assume

$$\begin{cases} \mu = 1; \quad C_0 = (\sqrt{T_0} + 3\sqrt{2})\beta_2 : \\ \sum_{i=1}^{3} k_i \le h = C_0\sqrt{T_0} = 1. \end{cases} \qquad (3)^*$$

And in this case all results of the theorem 6* are carried out, i.e. that is we will prove that under condition of (3)* system (5.25) correct in $\tilde{C}^{3,1}(T)$.

Really, as in case of (3)*, the interval $[0, T_0]$ we will divide on two parts: $[0, \frac{1}{2}T_0]$, $[\frac{1}{2}T_0, T_0]$. Thus a step: $h = \frac{1}{2}T_0$. Then in area $T_1 = R^3 \times [0, \frac{1}{2}T_0]$ and in $T_2 = R^3 \times [\frac{1}{2}T_0, T_0]$, we will receive systems

$$\begin{cases} Z_1 = \Gamma_{0,1}[Z_1, W_{1,(1)}, W_{2,(1)}, W_{3,(1)}], \forall (x_1, x_2, x_3, t) \in T_1, \\ W_{i,(1)} = \Gamma_{i,1}[Z_1, W_{1,(1)}, W_{2,(1)}, W_{3,(1)}], \forall (x_1, x_2, x_3, t) \in T_1, (i = \overline{1,3}). \end{cases} \qquad (4)^*$$

Thus, operators $\Gamma_{0,1}$; $\Gamma_{i,1}$ are compressing

$$d_0 = \sum_{i=1}^{3} k_i \le h_* = (\sqrt{\frac{T_0}{2}} + 3\sqrt{2})\beta_2 \frac{T_0}{2} < 1, \qquad (5)^*$$

and display ranges of definition in it self, i.e. conditions of contraction mapping principle [13] are satisfied. Then is under condition of (5)* system (4)* correct in $\tilde{C}^{3,1}(T_1)$.

Further, we will consider $T_2 = R^3 \times [\frac{1}{2}T_0, T_0]$, accordingly $\Gamma_{0,2}$; $\Gamma_{i,2}$, at that

$$\begin{cases} Z_1(x_1, x_2, x_3, \frac{1}{2}T_0) = Z_2(x_1, x_2, x_3, \frac{1}{2}T_0), \ \forall (x_1, x_2, x_3) \in R^3, \\ W_{i,(1)}(x_1, x_2, x_3, \frac{1}{2}T_0) = W_{i,(2)}(x_1, x_2, x_3, \frac{1}{2}T_0), \ \forall (x_1, x_2, x_3) \in R^3, (i = \overline{1,3}). \end{cases} \qquad (6)^*$$

Hence, operators $\Gamma_{0,2}$; $\Gamma_{i,2}$ are compressing and display ranges of definition in it self, i.e. conditions of contraction mapping principle are satisfied. Then system of the integral equations corresponding tothe operator $\Gamma_{0,2};\ \Gamma_{i,2}$ it is correct in $\tilde{C}^{3,1}(T_2)$.

Therefore the system (5.25) correct in $\tilde{C}^{3,1}(T = R^3 \times [0,T_0])$. The offered method the decision of system (5.25) in the theory of the Volterra's equations is called [13] «a method pasting» or «a method of subareas».

5.3. Fluid with Average and with a Great Number of Viscosity, when $f_i \equiv 0, (i=\overline{1,3})$

Area of the fluid with average viscosity with condition (A_3), when $f_i \geq 0, (i=\overline{1,3})$ it is studied in 5.1. Here we will consider a the general methods of the equations integration of Navier-Stokes with conditions, i.e. with average and with Reynolds small number ([12]: **Re**<2300), where all inertial participants contain in equations Navier-Stokes, when $f_i \equiv 0, (i=\overline{1,3})$.

The overall objective of this paragraph: to change a method (5.2) so that the received analytical decision of a problem Navier-Stokes with viscosity, belonged in $\tilde{C}^{3,1}_{n=3}(T)$.

In particular in many scientific literatures, investigates a problem [12]:

$$\frac{\partial \upsilon_i}{\partial t} + \sum_{j=1}^{3} \upsilon_j \frac{\partial \upsilon_i}{\partial x_j} = \mu \Delta \upsilon_i - \frac{1}{\rho} P_{x_i}, (i=\overline{1,3}), \tag{5.31}$$

$$\operatorname{div} \nu = 0, \forall (x,t) \in T = R^3 \times [0,T_0], \tag{5.32}$$

$$\begin{cases} \upsilon_i \big|_{t=0} = \lambda_i \vartheta_0(x_1,x_2,x_3), \forall (x_1,x_2,x_3) \in R^3, \\ \upsilon_{i0} \equiv \lambda_i \vartheta_0, \ (\vartheta_0 \in R^3;\ i=\overline{1,3}), \end{cases} \tag{5.33}$$

where $0 < \lambda_i -$ the known constants and in the equations (5.31) are assumed

$$f_i \equiv 0, (i=\overline{1,3};\ 0 < n_0 \leq \mu = \mu_0 < \infty;\ n_0, \mu_0 = \mathrm{const}). \tag{5.34}$$

Hence, here we will consider a methods of the equations integration of Navier-Stokes (5.31) with a conditions (5.32) - (5.34).

With that end in view we will assume that there are functions $0 \leq f_{i\delta}, (i=\overline{1,3})$, which satisfy conditions

$$\begin{cases} C^{3,0}(T) \ni f_{i\delta}: \ \sup_T \left| D^k f_{i\delta} \right| \leq \alpha_i(\delta) \leq \alpha(\delta) < 1, \ (0 < \delta < 1;\ i=\overline{1,3}), \\ \operatorname{div} f_\delta = 0;\ \operatorname{rot} f_\delta = 0:\ \Delta f_{i\delta} = 0,\ (f_\delta = (f_{1\delta}, f_{2\delta}, f_{3\delta})), \\ I_{i\delta}(x_1,x_2,x_3,t) \equiv \int_0^t f_{i\delta}(x_1,x_2,x_3,s')ds',\ (\Delta I_{i\delta} = 0;\ I_{i\delta t} \equiv f_{i\delta};\ i=\overline{1,3}), \\ \vartheta_0 \in R^3, \Delta \vartheta_0 = 0,\ (i=\overline{1,3}). \end{cases} \tag{5.35}$$

Hence is offered the method:

$$\begin{cases} \upsilon_i = \lambda_i[\vartheta_0 + Z(x_1, x_2, x_3, t)] + I_{i\delta}(x_1, x_2, x_3, t), (i = \overline{1,3}), \\ Z\big|_{t=0} = 0, \forall (x_1, x_2, x_3) \in R^3, \\ \operatorname{div} v = 0: \sum_{i=1}^{3} \lambda_i \vartheta_{0x_i} = 0; \quad \sum_{i=1}^{3} \lambda_i Z_{x_i} = 0; \quad \sum_{i=1}^{3} I_{i\delta x_i} = 0, \\ U = \sum_{j=1}^{3} I_{j\delta}^2; \quad \sum_{j=1}^{3} I_{j\delta} I_{i\delta x_j} = \frac{1}{2} U_{\delta x_i}. \end{cases} \tag{5.36}$$

Thus takes place conditions

$$\begin{cases} \sum_{i=1}^{3} \frac{\partial}{\partial x_i} I_{i\delta t} = 0, \\ \sum_{j=1}^{3} \upsilon_j \upsilon_{ix_j} \equiv \sum_{j=1}^{3} \lambda_j(\vartheta_0 + Z) I_{i\delta x_j} + \sum_{j=1}^{3} I_{j\delta} \lambda_i (\vartheta_{0x_j} + Z_{x_j}) + \frac{1}{2} U_{\delta x_i}, \\ \sum_{j=1}^{3} \lambda_j (\vartheta_0 + Z) \lambda_i (\vartheta_{0x_j} + Z_{x_j}) = \lambda_i (\vartheta_0 + Z) \sum_{j=1}^{3} \lambda_j (\vartheta_{0x_j} + Z_{x_j}) = 0, \\ \upsilon_{it} \equiv \lambda_i Z_t + I_{i\delta t}; \quad \mu \Delta \upsilon_i \equiv \mu[\lambda_i(\Delta \vartheta_0 + \Delta Z) + \Delta I_{i\delta}] = \mu \lambda_i \Delta Z, (i = \overline{1,3}). \end{cases} \tag{5.37}$$

Then for incompressible currents with a friction the equations of Navier-Stokes (5.31) become simpler as take place (5.32)-(5.34). Therefore the problem (5.31) - (5.33), is led to a kind

$$\lambda_i Z_t + \sum_{j=1}^{3} \lambda_j (\vartheta_0 + Z) I_{i\delta x_j} + \sum_{j=1}^{3} I_{j\delta} \lambda_i (\vartheta_{0x_j} + Z_{x_j}) + \frac{1}{2} U_{\delta x_i} = -f_{i\delta} - \frac{1}{\rho} P_{x_i} + \mu \lambda_i \Delta Z, (i = \overline{1,3}). \tag{5.38}$$

From system (5.38), considering conditions (5.31)-(5.33) and having entered APS, we will receive the equation:

$$\begin{cases} \sum_{i=1}^{3} \frac{\partial}{\partial x_i}(5.38): \quad \Delta\left(\frac{1}{\rho} P + \frac{1}{2} U_\delta\right) = -\{F_{0\delta} + B_\delta[Z_{x_1}, Z_{x_2}, Z_{x_3}]\}, \\ (B_\delta[Z_{x_1}, Z_{x_2}, Z_{x_3}])(x_1, x_2, x_3, t) \equiv \sum_{i=1}^{3} \left(\sum_{j=1}^{3} \lambda_j I_{i\delta x_j}\right) Z_{x_i} + \sum_{i=1}^{3} \left(\sum_{j=1}^{3} I_{j\delta x_i} Z_{x_j}\right) \lambda_i, \\ F_{0\delta}(x_1, x_2, x_3, t) \equiv \sum_{i=1}^{3} \lambda_i \left(\sum_{j=1}^{3} I_{j\delta x_i} \vartheta_{0x_j}\right) + \sum_{i=1}^{3} \vartheta_{0x_i} \left(\sum_{j=1}^{3} \lambda_j I_{i\delta x_j}\right); \ \operatorname{div} f_\delta = 0, \\ \frac{1}{\rho} P + \frac{1}{2} U_\delta = \frac{1}{4\pi} \int_{R^3} \frac{1}{r} \{F_{0\delta}(s_1, s_2, s_3, t) + (B_\delta[Z_{s_1}, Z_{s_2}, Z_{s_3}])(s_1, s_2, s_3, t)\} ds_1 ds_2 ds_3, \\ \frac{1}{\rho} P_{x_i} + \frac{1}{2} U_{\delta x_i} = \frac{1}{4\pi} \int_{R^3} \frac{\tau_i}{\sqrt{(\tau_1^2 + \tau_2^2 + \tau_3^2)^3}} \{F_{0\delta}(x_1 + \tau_1, x_2 + \tau_2, x_3 + \tau_3; t) + \\ + (B_\delta[Z_{h_1}, Z_{h_2}, Z_{h_3}])(x_1 + \tau_1, x_2 + \tau_2, x_3 + \tau_3; t)\} d\tau_1 d\tau_2 d\tau_3, (s_i - x_i = \tau_i; h_i = x_i + \tau_i; i = \overline{1,3}), \end{cases} \tag{5.39}$$

so as takes place

$$\begin{cases} \sum_{i=1}^{3} \frac{\partial}{\partial x_i} \{\sum_{j=1}^{3} \lambda_j (\vartheta_0 + Z) I_{i\delta x_j} + \sum_{j=1}^{3} I_{j\delta} \lambda_i (\vartheta_{0x_j} + Z_{x_j})\} = \sum_{i=1}^{3} \lambda_i \left(\sum_{j=1}^{3} I_{j\delta x_i} \vartheta_{0x_j}\right) + \sum_{i=1}^{3} \vartheta_{0x_i} \left(\sum_{j=1}^{3} \lambda_j I_{i\delta x_j}\right) + \\ + \sum_{i=1}^{3} \left(\sum_{j=1}^{3} \lambda_j I_{i\delta x_j}\right) Z_{x_i} + \sum_{i=1}^{3} \left(\sum_{j=1}^{3} I_{j\delta x_i} Z_{x_j}\right) \lambda_i, \end{cases}$$

$$
\begin{cases}
(\sum_{j=1}^{3} I_{j\delta x_j})_{x_i} = 0;\ (\sum_{j=1}^{3}\lambda_j\vartheta_{0x_j})_{x_i} = 0,\ (\sum_{j=1}^{3}\lambda_j Z_{x_j})_{x_i} = 0,\ (i=\overline{1,3}),\\
\frac{\partial}{\partial t}[\sum_{i=1}^{3}\lambda_i Z_{x_i}] = 0;\ \ \mu\sum_{i=1}^{3}\frac{\partial}{\partial x_i}(\lambda_i\Delta Z) = 0;\ \ \sum_{i=1}^{3}\frac{\partial}{\partial x_i}(\frac{1}{2}U_{\delta x_i}) = \frac{1}{2}\Delta U_\delta;\ \ \sum_{i=1}^{3}\frac{\partial}{\partial x_i}(-\frac{1}{\rho}P_{x_i}) \equiv -\frac{1}{\rho}\Delta P.
\end{cases}
$$

Then on a basis (5.39) of system (5.38) it is equivalent, will be transformed to a kind:

$$
\begin{cases}
Z_t = \Phi_{0\delta} + (Q[Z,Z_{x_1},Z_{x_2},Z_{x_3}])(x_1,x_2,x_3,t) + \mu\Delta Z,(i=\overline{1,3}),\\
Z\big|_{t=0} = 0,\forall(x_1,x_2,x_3)\in R^3,\\
\Phi_{0\delta}(x_1,x_2,x_3,t) \equiv -\sum_{j=1}^{3} I_{j\delta}\vartheta_{0x_j} + d_0^{-1}[\sum_{i=1}^{3}(-f_{i\delta}) - \sum_{i=1}^{3}\vartheta_0(\sum_{j=1}^{3}\lambda_j I_{i\delta x_j}) -\\
-\frac{1}{4\pi}\int_{R^3}\{\sum_{i=1}^{3}\frac{\tau_i}{\sqrt{(\tau_1^2+\tau_2^2+\tau_3^2)^3}}(F_{0\delta}(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t))\}d\tau_1 d\tau_2 d\tau_3],\\
(Q[Z,Z_{x_1},Z_{x_2},Z_{x_3}])(x_1,x_2,x_3,t) \equiv -\{d_0^{-1}Z(x_1,x_2,x_3,t)[\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j I_{i\delta x_j}(x_1,x_2,x_3,t))] +\\
+\sum_{j=1}^{3} Z_{x_j}(x_1,x_2,x_3,t)I_{j\delta}(x_1,x_2,x_3,t) + d_0^{-1}[\frac{1}{4\pi}\int_{R^3}(\sum_{i=1}^{3}\frac{\overline{\tau}_i}{\sqrt{(\overline{\tau}_1^2+\overline{\tau}_2^2+\overline{\tau}_3^2)^3}}\{(B_\delta[Z_{\overline{h}_1},Z_{\overline{h}_2},Z_{\overline{h}_3}])(x_1+\\
+\overline{\tau}_1,x_2+\overline{\tau}_2,x_3+\overline{\tau}_3;t)\})d\overline{\tau}_1 d\overline{\tau}_2 d\overline{\tau}_3]\},\\
d_0 = \sum_{i=1}^{3}\lambda_i > 0;\ \ \overline{h}_1 = x_i + \overline{\tau}_i,\ \ (i=\overline{1,3}).
\end{cases}
\tag{5.40}
$$

The problem (5.40) is led to system of the integrated equations in a kind

$$
\begin{cases}
Z_{x_i} = W_i(x_1,x_2,x_3,t),\forall(x_1,x_2,x_3,t)\in T,(i=\overline{1,3}),\\
Z = M_{1\delta} + \frac{1}{8\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-\frac{r^2}{4\mu(t-s)})\frac{1}{(\sqrt{\mu(t-s)})^3}(Q[Z,W_1,W_2,W_3])(s_1,s_2,s_3,s)\times\\
\times ds_1 ds_2 ds_3 ds = M_{1\delta} + \frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))(Q[Z,W_1,W_2,W_3])(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+\\
+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)d\tau_1 d\tau_2 d\tau_3 ds \equiv (\Gamma_0[Z,W_1,W_2,W_3])(x_1,x_2,x_3,t),\\
W_i = M_{1\delta x_i} + \frac{1}{8\sqrt{\pi^3}}\int_0^t\int_{R^3}(\exp(-\frac{r^2}{4\mu(t-s)}))\frac{-(x_i-s_i)}{2\mu(t-s)}\frac{1}{(\sqrt{\mu(t-s)})^3}(Q[Z,W_1,W_2,W_3])\times\\
\times(s_1,s_2,s_3,s)ds_1 ds_2 ds_3 ds = M_{1\delta x_i} + \frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\frac{\tau_i}{\sqrt{\mu(t-s)}}\times\\
\times(Q[Z,W_1,W_2,W_3])(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)\times\\
\times d\tau_1 d\tau_2 d\tau_3 ds \equiv (\Gamma_i[Z,W_1,W_2,W_3])(x_1,x_2,x_3,t),\ (s_i - x_i = 2\tau_i\sqrt{\mu(t-s)};i=\overline{1,3}),\\
M_{1\delta}(x_1,x_2,x_3,t) \equiv \frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Phi_{0\delta}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\times\\
\times\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)d\tau_1 d\tau_2 d\tau_3 ds.
\end{cases}
\tag{5.41}
$$

To solve a problem (5.40) concerning this problem we will receive system (5.41) of four integral equations.

Let concerning known functions $M_{1\delta}, \Pi_{\delta}, I_{i\delta}$ takes place:

$$\begin{cases} \forall (x_1,x_2,x_3,t)\in T;\, M_{1\delta},\Pi_{\delta},I_{i\delta}: \\ \sup\limits_{T}\left|D^k M_{1\delta}(x_1,x_2,x_3,t)\right| \le \beta_1\alpha(\delta),\ (k=\overline{0,3};\ \ t-s=\tau), \\ \sup\limits_{T\times T}\Pi_{\delta}(x_1,x_2,x_3,\tau_1,\tau_2,\tau_3;t,\tau)\equiv \sup\limits_{T\times T}\{d_0^{-1}\sum\limits_{i=1}^{3}(\sum\limits_{j=1}^{3}\lambda_j\left|I_{i\delta s_j}(x_1+2\tau_1\sqrt{\mu\tau},x_2+2\tau_2\sqrt{\mu\tau},x_3+2\tau_3\times\right. \\ \left.\times\sqrt{\mu\tau};t-\tau)\right|)+\sum\limits_{j=1}^{3}\left|I_{j\delta}(x_1+2\tau_1\sqrt{\mu\tau},x_2+2\tau_2\sqrt{\mu\tau},x_3+2\tau_3\sqrt{\mu\tau};t-\tau)\right|+ \\ +d_0^{-1}[\frac{1}{4\pi}\int\limits_{R^3}(\sum\limits_{i=1}^{3}\frac{\left|\overline{\tau}_i\right|}{\sqrt{(\overline{\tau}_1^2+\overline{\tau}_2^2+\overline{\tau}_3^2)^3}}\{\sum\limits_{i=1}^{3}(\sum\limits_{j=1}^{3}\lambda_j\left|I_{i\delta l_j}(x_1+2\tau_1\sqrt{\mu\tau}+\overline{\tau}_1,x_2+2\tau_2\sqrt{\mu\tau}+\overline{\tau}_2,x_3+\right. \\ \left.+2\tau_3\sqrt{\mu\tau}+\overline{\tau}_3;t-\tau)\right|)+\sum\limits_{i=1}^{3}(\sum\limits_{j=1}^{3}\left|I_{j\delta l_i}(x_1+2\tau_1\sqrt{\mu\tau}+\overline{\tau}_1,x_2+2\tau_2\sqrt{\mu\tau}+\overline{\tau}_2,x_3+2\tau_3\sqrt{\mu\tau}+\right. \\ \left.+\overline{\tau}_3;t-\tau)\right|)\lambda_i\})d\overline{\tau}_1d\overline{\tau}_2d\overline{\tau}_3]\}\le\beta_2\alpha(\delta), \\ k_0=\frac{1}{\sqrt{\pi^3}}\sup\limits_{T}\int\limits_0^t\int\limits_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Pi_{\delta}(x_1,x_2,x_3,\tau_1,\tau_2,\tau_3;t,\tau)d\tau_1d\tau_2d\tau_3d\tau\le\beta_2\alpha(\delta)T_0, \\ k_i=\frac{1}{\sqrt{\pi^3}}\sup\limits_{T}\int\limits_0^t\int\limits_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Pi_{\delta}(x_1,x_2,x_3,\tau_1,\tau_2,\tau_3;t,\tau)\frac{\left|\tau_i\right|}{\sqrt{\mu\tau}}d\tau_1d\tau_2d\tau_3d\tau\le \\ \le\sqrt{2T_0}\beta_2\alpha(\delta)\frac{1}{\sqrt{\mu}},(i=\overline{1,3}),\ \beta=\max(\beta_2T_0;\ 3\sqrt{2T_0}\beta_2), \end{cases} \tag{5.42}$$

and if operators: $\Gamma_i,(i=\overline{0,3})$ compressing with a compression factor k_i,

$$\begin{cases} \Gamma_i,(i=\overline{0,3}):\ k_i\le\frac{h}{4},(h<1),\ (i=\overline{1,3}), \\ \sum\limits_{i=0}^{3}k_i\le\alpha(\delta)(\beta_2T_0\frac{1}{\sqrt{\mu}}+3\sqrt{2T_0}\beta_2)\le\alpha(\delta)(\frac{1}{\sqrt{n_0}}+1)\beta=h<1, \\ \alpha(\delta)<\delta[(\frac{1}{\sqrt{n_0}}+1)\beta]^{-1},\ (0<\delta<1;\ \ 0<n_0\le\mu=\mu_0<\infty), \\ S_{r_1}(0)=\{Z,W_i:\left|Z\right|;\ \left|W_i\right|\le r_1,\forall(x_1,x_2,x_3,t)\in T\},(i=\overline{1,3}), \end{cases} \tag{5.43}$$

and:

$$\begin{cases} \left\|\Gamma_i[0,0,0,0]\right\|_C\le r_1(1-h): \\ \left\|\Gamma_i[Z,W_1,W_2,W_3]\right\|_C\le\left\|\Gamma_i[Z,W_1,W_2,W_3]-\Gamma_i[0,0,0,0]\right\|_C+\left\|\Gamma_i[0,0,0,0]\right\|_C\le \\ \le k_i4r_1+r_1(1-h)\le hr_1+r_1(1-h)=r_1, \\ \Gamma_i:S_{r_1}(0)\to S_{r_1}(0),\ (i=\overline{0,3}). \end{cases} \tag{5.44}$$

Hence on the basis of contraction mapping principle system (5.41) is solvable and for which makes Picard's method

$$\begin{cases} \forall (x_1,x_2,x_3,t) \in T: \quad Z_{n+1} = \Gamma_0[Z_n, W_{1,n}, W_{2,n}, W_{3,n}], \\ W_{i,n+1} = \Gamma_i[Z_n, W_{1,n}, W_{2,n}, W_{3,n}], (n = 0,1,\ldots; Z_0 = 0; \quad W_{i,0} = 0;\ i = \overline{1,3}). \end{cases} \tag{5.45}$$

The resulting sequence of functions $\{Z_n\}_0^\infty, \{W_{i,n}\}_0^\infty$ is converging and fundamental in $S_{r_1}(0)$, i.e.:

$$\begin{cases} E_{n+1} = \|Z_{n+1} - Z_n\|_C + \sum_{i=1}^{3} \|W_{i,n+1} - W_{i,n}\|_C;\ E_n = \|Z_n - Z_{n-1}\|_C + \sum_{i=1}^{3} \|W_{i,n} - W_{i,n-1}\|_C: \\ \|Z_{n+1} - Z_n\|_C \le k_0 E_n; \quad \|W_{i,n+1} - W_{i,n}\|_C \le k_i E_n, (i = \overline{1,3}), \\ E_{n+1} \le hE_n \le \ldots \le h^n E_1 \xrightarrow[n\to\infty]{h<1} 0, \\ \|Z_{n+k} - Z_n\|_C \le \sum_{j=0}^{k-1} k_0 E_{n+j}; \quad \|W_{i,n+k} - W_{i,n}\|_C \le \sum_{j=0}^{k-1} k_i E_{n+j}, (i = \overline{1,3}), \\ E_{n+k} \le h\sum_{j=0}^{k-1} E_{n+j} \le \ldots \le h\sum_{j=0}^{k-1} h^{n+j-1}E_1 \le E_1 h^n \sum_{j=0}^{k-1} h^j \le E_1 h^n \dfrac{1}{1-h} \xrightarrow[n\to\infty]{h<1} 0, \\ X_0 = \|Z\|_C + \sum_{i=1}^{3} \|W_i\|_C;\ X_{n+1} \equiv \|Z_{n+1} - Z\|_C + \sum_{i=1}^{3} \|W_{i,n+1} - W_i\|_C: \\ X_{n+1} \le h^{n+1} X_0 \xrightarrow[n\to\infty]{h<1} 0, \\ Z_{n+1} \xrightarrow[n\to\infty]{h<1} Z \equiv H; \quad W_{i,n+1} \xrightarrow[n\to\infty]{h<1} W_i, \forall (x_1,x_2,x_3,t) \in T, (i = \overline{1,3}). \end{cases} \tag{5.46}$$

Therefore on the basis of (5.36) and

$$\upsilon_{i,n+1} = \lambda_i[\vartheta_0(x_1,x_2,x_3) + Z_{n+1}(x_1,x_2,x_3,t)] + I_{i\delta}(x_1,x_2,x_3,t), (n = 0,1,2,\ldots; i = \overline{1,3}), \tag{5.47}$$

we will receive

$$\|\upsilon_{i,n+1} - \upsilon_i\|_C \le \lambda_i \|Z_{n+1} - Z\|_C \le \lambda_i h X_n \le \lambda_i h^{n+1} X_0 \xrightarrow[n\to\infty]{h<1} 0, (i = \overline{1,3}).$$

And it means that sequence $\{\upsilon_{i,n}\}_0^\infty$ converging to a limit $\upsilon_i, (i = \overline{1,3}):$

$$\upsilon_{i,n+1} \xrightarrow[n\to\infty]{h<1} \upsilon_i \in \tilde{C}^{3,1}(T), (i = \overline{1,3}). \tag{5.48}$$

Remark 4. It is obvious that small changes $\upsilon_{i0} \equiv \lambda_i \vartheta_0$ or $f_{i\delta}, (i = \overline{1,3})$ influence the decision (5.36) a little, i.e. continuous depends on this data . Therefore, a question on a statement correctness problems (5.31)-(5.33) are considered at once with results of the theorem 6* in $\tilde{C}^{3,1}_{n=3}(T)$.

6. Updating of a Method (4.12) on the Basis of Integrals of Poisson’s Type

In the previous paragraphs we researched various variants of a method (4.12) for a problem Navier-Stokes in spaces $\tilde{C}^{3,1}_{n=3}(T)$. There are the various partial experimental methods [12] giving communication of speed and pressure. For example, method Betz, A., Jones B.M. and others, where on the basis of the law of distribution of pressure in the form of the equation of Bernoulli it is possible to express to speed $v=(\upsilon_1,\upsilon_2,\upsilon_3)$ in the certain form. Therefore, here similar results which define communication between pressure and speeds are received, and further, allow to express speed in the integral form.

In this connection in this paragraph the method of integrated transformation on the basis of Poisson's integrals is considered, when $\mathrm{div} f=0;\ 0<\mu<1$ (Reynolds number [12]: **Re** ≥2300). From the received results it follows that system Navier-Stokes (1.1) in the conditions of (1.2), (1.3), (A_3) can have the analytical smooth single solution in $\tilde{C}^{3,1}_{n=3}(T)$.

6.1. Fluid with Very Small Viscosity $0<\mu<1$, when $\mathrm{div} f=0$

I. For incompressible currents with a friction, when

$$\begin{cases} \upsilon_i\big|_{t=0}=\upsilon_{i0}(x_1,x_2,x_3)\equiv\lambda_i\vartheta_0(x_1,x_2,x_3),(i=\overline{1,3}), \\ \sum\limits_{i=1}^{3}\lambda_i\vartheta_{0x_i}=0;\quad \Delta\vartheta_0=0;\ \vartheta_0\in R^3, \\ \sup\limits_{T}\left|D^k f_i\right|\le N_0=\mathrm{const},\ (T=R^3\times[0,T_0]),\ (i=\overline{1,3}), \\ \mathrm{div} f=0;\ \ r=\sqrt{(x_1-s_1)^2+(x_2-s_2)^2+(x_3-s_3)^2}, \\ \Omega_i(x_1,x_2,x_3,t)\equiv\mu\dfrac{1}{8\sqrt{\pi^3}}\int\limits_0^t\int\limits_{R^3}\exp\left(-\dfrac{r^2}{4\mu(t-s)}\right)\dfrac{1}{\sqrt{(\mu(t-s))^3}}f_i(s_1,s_2,s_3,s)ds_1ds_2ds_3ds= \\ =\mu\dfrac{1}{\sqrt{\pi^3}}\int\limits_0^t\int\limits_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))f_i(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\times \\ \times\sqrt{\mu(t-s)};s)d\tau_1d\tau_2d\tau_3ds, \\ s_i-x_i=2\tau_i\sqrt{\mu(t-s)},\ (i=\overline{1,3}), \end{cases} \tag{6.1}$$

functions $\upsilon_i, i=1,2,3$ is represented in a kind

$$\begin{cases} \upsilon_i=\lambda_i[\vartheta_0(x_1,x_2,x_3)+Z(x_1,x_2,x_3,t)]+\Omega_i(x_1,x_2,x_3,t),\forall(x_1,x_2,x_3,t)\in T,(i=\overline{1,3}), \\ Z\big|_{t=0}=0,\ \ \forall(x_1,x_2,x_3)\in R^3, \end{cases} \tag{6.2}$$

where $0<\lambda_i -$ the known constants.

From the entered functions Ω_i, is required:

$$\begin{cases} \lim\limits_{\mu\to 0}\Omega_i(x_1,x_2,x_3,t)=0,\forall(x_1,x_2,x_3,t)\in T, \\ \Omega_{it}\equiv\mu f_i(x_1,x_2,x_3,t)+\mu\dfrac{1}{\sqrt{\pi^3}}\int\limits_0^t\int\limits_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))[\sum\limits_{j=1}^{3}\dfrac{\tau_j\sqrt{\mu}}{\sqrt{t-s}}f_{il_j}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+ \\ +2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)]d\tau_1d\tau_2d\tau_3ds,(i=\overline{1,3}), \end{cases}$$

$$
\left\{
\begin{array}{l}
\Omega_{ix_j} = \mu \dfrac{1}{8\sqrt{\pi^3}} \displaystyle\int_0^t \int_{R^3} \dfrac{-(x_j - s_j)}{2\mu(t-s)} \dfrac{1}{(\sqrt{\mu(t-s)})^3} \exp\left(-\dfrac{r^2}{4\mu(t-s)}\right) f_i(s_1, s_2, s_3, s)\, ds_1 ds_2 ds_3 ds = \\
= \sqrt{\mu} \dfrac{1}{\sqrt{\pi^3}} \displaystyle\int_0^t \int_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2)) \dfrac{\tau_j}{\sqrt{t-s}} f_i(x_1 + 2\tau_1\sqrt{\mu(t-s)}, x_2 + 2\tau_2\sqrt{\mu(t-s)}, x_3 + \\
+ 2\tau_3\sqrt{\mu(t-s)}; s)\, d\tau_1 d\tau_2 d\tau_3 ds, (i = \overline{1,3};\ j = \overline{1,3}), \\
\Omega_{ix_j^2} = \sqrt{\mu} \dfrac{1}{\sqrt{\pi^3}} \displaystyle\int_0^t \int_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2)) \dfrac{\tau_j}{\sqrt{t-s}} f_{il_j}(x_1 + 2\tau_1\sqrt{\mu(t-s)}, x_2 + 2\tau_2\sqrt{\mu(t-s)}, \\
x_3 + 2\tau_3\sqrt{\mu(t-s)}; s)\, d\tau_1 d\tau_2 d\tau_3 ds, \\
\Omega_{ix_j^3} = \sqrt{\mu} \dfrac{1}{\sqrt{\pi^3}} \displaystyle\int_0^t \int_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2)) \dfrac{\tau_j}{\sqrt{t-s}} f_{il_j^2}(x_1 + 2\tau_1\sqrt{\mu(t-s)}, x_2 + 2\tau_2\sqrt{\mu(t-s)}, \\
x_3 + 2\tau_3\sqrt{\mu(t-s)}; s)\, d\tau_1 d\tau_2 d\tau_3 ds, (i = \overline{1,3};\ j = \overline{1,3}), \\
\left|\Omega_i\right| \equiv \left|\mu \displaystyle\int_0^t \tilde{P}_i(x_1, x_2, x_3, t, s)\, ds\right| \equiv \dfrac{1}{\sqrt{\pi^3}} \left| \displaystyle\int_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2)) f_i(x_1 + 2\tau_1\sqrt{\mu(t-s)}, x_2 + \right. \\
\left. + 2\tau_2\sqrt{\mu(t-s)}, x_3 + 2\tau_3\sqrt{\mu(t-s)}; s)\, d\tau_1 d\tau_2\, d\tau_3 \right| \le N_0 \dfrac{1}{\sqrt{\pi^3}} \displaystyle\int_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2)) \times \\
\times d\tau_1 d\tau_2 d\tau_3 = N_0, (i = \overline{1,3}),
\end{array}
\right.
$$

here $(0,1) \in \mu$ in a role of small parameter. At that

$$
\left\{
\begin{array}{l}
\mathrm{div}\, v = 0 : \displaystyle\sum_{i=1}^3 \lambda_i Z_{x_i} = 0;\quad \sum_{i=1}^3 \lambda_i \vartheta_{0x_i} = 0;\quad \sum_{i=1}^3 \Omega_{ix_i} = 0, \\
\mu\Delta\Omega_i = \mu\sqrt{\mu} \dfrac{1}{\sqrt{\pi^3}} \displaystyle\int_0^t \int_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2))[\sum_{j=1}^3 \dfrac{\tau_j}{\sqrt{t-s}} f_{il_j}(x_1 + 2\tau_1\sqrt{\mu(t-s)}, x_2 + \\
+ 2\tau_2\sqrt{\mu(t-s)}, x_3 + 2\tau_3\sqrt{\mu(t-s)}; s)]\, d\tau_1 d\tau_2 d\tau_3 ds, \\
l_j = x_j + 2\tau_j\sqrt{\mu(t-s)}, (i = \overline{1,3};\ j = \overline{1,3}), \\
\displaystyle\sum_{j=1}^3 \upsilon_j \upsilon_{ix_j} \equiv \sum_{j=1}^3 \lambda_j(\vartheta_0 + Z)\Omega_{ix_j} + \sum_{j=1}^3 \Omega_j \lambda_i(\vartheta_{0x_j} + Z_{x_j}) + \sum_{j=1}^3 \Omega_j \Omega_{ix_j}, \\
\displaystyle\sum_{j=1}^3 \lambda_j(\vartheta_0 + Z)\lambda_i(\vartheta_{0x_j} + Z_{x_j}) = \lambda_i(\vartheta_0 + Z)\sum_{j=1}^3 \lambda_j(\vartheta_{0x_j} + Z_{x_j}) = 0, \\
\upsilon_{it} \equiv \lambda_i Z_t + \Omega_{it};\quad \mu\Delta\upsilon_i \equiv \mu[\lambda_i \Delta Z + \Delta\Omega_i],\ (\Delta\vartheta_0 = 0;\quad i = \overline{1,3}), \\
\upsilon_{it} - \mu\Delta\upsilon_i \equiv \lambda_i Z_t + \mu f_i - \mu\sqrt{\mu} \dfrac{1}{\sqrt{\pi^3}} \displaystyle\int_0^t \int_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2))[\sum_{j=1}^3 \dfrac{\tau_j}{\sqrt{t-s}} f_{il_j}(x_1 + \\
+ 2\tau_1\sqrt{\mu(t-s)}, x_2 + 2\tau_2\sqrt{\mu(t-s)}, x_3 + 2\tau_3\sqrt{\mu(t-s)}; s)]\, d\tau_1 d\tau_2 d\tau_3 ds - \mu\{\lambda_i \Delta Z + \\
+ \sqrt{\mu} \dfrac{1}{\sqrt{\pi^3}} \displaystyle\int_0^t \int_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2))[\sum_{j=1}^3 \dfrac{\tau_j}{\sqrt{t-s}} f_{il_j}(x_1 + 2\tau_1\sqrt{\mu(t-s)}, x_2 + 2\tau_2 \times \\
\times \sqrt{\mu(t-s)}, x_3 + 2\tau_3\sqrt{\mu(t-s)}; s)]\, d\tau_1 d\tau_2 d\tau_3 ds\} = \lambda_i Z_t + \mu f_i - \mu\lambda_i \Delta Z,\ (i = \overline{1,3}).
\end{array}
\right.
\quad (6.3)
$$

Then for incompressible currents with a friction the equations of Navier-Stokes (1.1) become simpler as take place (6.1)-(6.3). Therefore the system (1.1), is led to a kind

$$\lambda_i Z_t + \sum_{j=1}^{3} \lambda_j(\vartheta_0 + Z)\Omega_{ix_j} + \sum_{j=1}^{3} \Omega_j \lambda_i(\vartheta_{0x_j} + Z_{x_j}) + \sum_{j=1}^{3} \Omega_j \Omega_{ix_j} = (1-\mu) f_i - \frac{1}{\rho} P_{x_i} + \tag{6.4}$$
$$+ \mu\lambda_i \Delta Z, \forall (x_1, x_2, x_3, t) \in T, (i = \overline{1,3}).$$

From system (6.4), considering conditions (6.1)-(6.3) and having entered APS we have the equation:

$$\begin{cases} \sum_{i=1}^{3} \frac{\partial}{\partial x_i}(6.4): \quad \Delta \frac{1}{\rho} P = -\{F_0 + \tilde{B}[Z_{x_1}, Z_{x_2}, Z_{x_3}]\}, \\ (\tilde{B}[Z_{x_1}, Z_{x_2}, Z_{x_3}])(x_1, x_2, x_3, t) \equiv \sum_{i=1}^{3} (\sum_{j=1}^{3} \lambda_j \Omega_{ix_j}) Z_{x_i} + \sum_{i=1}^{3} (\sum_{j=1}^{3} \Omega_{jx_i} Z_{x_j}) \lambda_i, \\ \mathrm{div} f = 0; \quad F_0(x_1, x_2, x_3, t) \equiv \sum_{i=1}^{3} \frac{\partial}{\partial x_i}(\sum_{j=1}^{3} \Omega_j \Omega_{ix_j}) + \sum_{i=1}^{3} \lambda_i (\sum_{j=1}^{3} \Omega_{jx_i} \vartheta_{0x_j}) + \sum_{i=1}^{3} \vartheta_{0x_i}(\sum_{j=1}^{3} \lambda_j \Omega_{ix_j}), \\ \frac{1}{\rho} P = \frac{1}{4\pi} \int_{R^3} \frac{1}{r}\{F_0(s_1, s_2, s_3, t) + (\tilde{B}[Z_{s_1}, Z_{s_2}, Z_{s_3}])(s_1, s_2, s_3, t)\} ds_1 ds_2 ds_3, \\ \frac{1}{\rho} P_{x_i} = \frac{1}{4\pi} \int_{R^3} \frac{\tau_i}{\sqrt{(\tau_1^2 + \tau_2^2 + \tau_3^2)^3}} \{F_0(x_1 + \tau_1, x_2 + \tau_2, x_3 + \tau_3; t) + (\tilde{B}[Z_{h_1}, Z_{h_2}, Z_{h_3}])(x_1 + \tau_1, x_2 + \\ + \tau_2, x_3 + \tau_3; t)\} d\tau_1 d\tau_2 d\tau_3, \quad (s_i - x_i = \tau_i; h_i = x_i + \tau_i; i = \overline{1,3}), \end{cases} \tag{6.5}$$

so as takes place

$$\begin{cases} \sum_{i=1}^{3} \frac{\partial}{\partial x_i}\{\sum_{j=1}^{3} \lambda_j(\vartheta_0 + Z)\Omega_{ix_j} + \sum_{j=1}^{3} \Omega_j \lambda_i(\vartheta_{0x_j} + Z_{x_j}) + \sum_{j=1}^{3} \Omega_j \Omega_{ix_j}\} = \sum_{i=1}^{3} \lambda_i (\sum_{j=1}^{3} \Omega_{jx_i} \vartheta_{0x_j}) + \\ + \sum_{i=1}^{3} \vartheta_{0x_i}(\sum_{j=1}^{3} \lambda_j \Omega_{ix_j}) + \sum_{i=1}^{3} (\sum_{j=1}^{3} \lambda_j \Omega_{ix_j}) Z_{x_i} + \sum_{i=1}^{3} (\sum_{j=1}^{3} \Omega_{jx_i} Z_{x_j}) \lambda_i, \\ (\sum_{j=1}^{3} \Omega_{jx_j})_{x_i} = 0; \ (\sum_{j=1}^{3} \lambda_j \vartheta_{0x_j})_{x_i} = 0, \ (\sum_{j=1}^{3} \lambda_j Z_{x_j})_{x_i} = 0, \ (i = \overline{1,3}), \\ \frac{\partial}{\partial t}[\sum_{i=1}^{3} \lambda_i Z_{x_i}] = 0; \quad \sum_{i=1}^{3} \frac{\partial}{\partial x_i}(-\frac{1}{\rho} P_{x_i}) \equiv -\frac{1}{\rho} \Delta P; \quad \mu \sum_{i=1}^{3} \frac{\partial}{\partial x_i}(\lambda_i \Delta Z) = 0. \end{cases}$$

Then on a basis (6.5) system (6.4) it is equivalent, will be transformed to a kind:

$$\begin{cases} Z_t = \Phi_0 + (Q[Z, Z_{x_1}, Z_{x_2}, Z_{x_3}])(x_1, x_2, x_3, t) + \mu \Delta Z, (i = \overline{1,3}), \\ Z\big|_{t=0} = 0, \forall (x_1, x_2, x_3) \in R^3, \end{cases} \tag{6.6}$$

where

$$\begin{cases} \Phi_0(x_1, x_2, x_3, t) \equiv -\sum_{j=1}^{3} \Omega_j \vartheta_{0x_j} + d_0^{-1}[\sum_{i=1}^{3} (1-\mu) f_i - \sum_{i=1}^{3} \vartheta_0(\sum_{j=1}^{3} \lambda_j \Omega_{ix_j}) - \sum_{i=1}^{3} (\sum_{j=1}^{3} \Omega_j \Omega_{ix_j}) - \\ - \frac{1}{4\pi} \int_{R^3} \{\sum_{i=1}^{3} \frac{\tau_i}{\sqrt{(\tau_1^2 + \tau_2^2 + \tau_3^2)^3}} (F_0(x_1 + \tau_1, x_2 + \tau_2, x_3 + \tau_3; t))\} d\tau_1 d\tau_2 d\tau_3]; \ d_0 = \sum_{i=1}^{3} \lambda_i > 0, \end{cases}$$

$$\begin{cases} (Q[Z,Z_{x_1},Z_{x_2},Z_{x_3}])(x_1,x_2,x_3,t) \equiv -\{d_0^{-1}Z(x_1,x_2,x_3,t)[\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j\Omega_{ix_j}(x_1,x_2,x_3,t))]+ \\ +\sum_{j=1}^{3}Z_{x_j}(x_1,x_2,x_3,t)\Omega_j(x_1,x_2,x_3,t)+d_0^{-1}[\frac{1}{4\pi}\int_{R^3}(\sum_{i=1}^{3}\frac{\overline{\tau}_i}{\sqrt{(\overline{\tau}_1^2+\overline{\tau}_2^2+\overline{\tau}_3^2)^3}}\{(\tilde{B}[Z_{l_1},Z_{l_2},Z_{l_3}])(x_1+ \\ +\overline{\tau}_1,x_2+\overline{\tau}_2,x_3+\overline{\tau}_3;t)\})d\overline{\tau}_1 d\overline{\tau}_2 d\overline{\tau}_3]\},\ (l_i=x_i+\overline{\tau}_i;\quad i=\overline{1,3}). \end{cases}$$

The problem (6.6) is led to system of the integrated equations quite regular rather $(0,1)\in\mu$, in a kind

$$\begin{cases} Z_{x_i}=W_i(x_1,x_2,x_3,t),\forall(x_1,x_2,x_3,t)\in T,(i=\overline{1,3}), \\ Z=M_1+\frac{1}{8\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-\frac{r^2}{4\mu(t-s)})\frac{1}{(\sqrt{\mu(t-s)})^3}(Q[Z,W_1,W_2,W_3])(s_1,s_2,s_3,s)\times \\ \times ds_1ds_2ds_3ds=M_1+\frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))(Q[Z,W_1,W_2,W_3])(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+ \\ +2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)d\tau_1d\tau_2d\tau_3ds\equiv(\Psi_0[Z,W_1,W_2,W_3])(x_1,x_2,x_3,t), \\ W_i=M_{1x_i}+\frac{1}{8\sqrt{\pi^3}}\int_0^t\int_{R^3}(\exp(-\frac{r^2}{4\mu(t-s)}))\frac{-(x_i-s_i)}{2\mu(t-s)}\frac{1}{(\sqrt{\mu(t-s)})^3}(Q[Z,W_1,W_2,W_3])\times \\ \times(s_1,s_2,s_3,s)ds_1ds_2ds_3ds=M_{1x_i}+\frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\frac{\tau_i}{\sqrt{\mu(t-s)}}\times \\ \times(Q[Z,W_1,W_2,W_3])(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)d\tau_1d\tau_2d\tau_3ds\equiv \\ \equiv(\Psi_i[Z,W_1,W_2,W_3])(x_1,x_2,x_3,t),\quad(s_i-x_i=2\tau_i\sqrt{\mu(t-s)};i=\overline{1,3}), \\ M_1(x_1,x_2,x_3,t)\equiv\frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Phi_0(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+ \\ +2\tau_3\sqrt{\mu(t-s)};s)d\tau_1d\tau_2d\tau_3ds. \end{cases} \tag{6.7}$$

Let's notice that if known functions f_i satisfy conditions of submultiplications [15] and takes place:

$$\begin{cases} \forall(x_1,x_2,x_3,t)\in T;M_1;\Pi;\Omega_i:\qquad \sup_T\left|D^kM_1(x_1,x_2,x_3,t)\right|\le\beta_1,\ (k=\overline{0,3};\ \ t-s=\tau), \\ \Omega_i\equiv\mu\frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))f_i(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\times \\ \times\sqrt{\mu(t-s)};s)d\tau_1d\tau_2d\tau_3ds,\ \ (i=\overline{1,3}), \\ \sup_{T\times T}\Pi(x_1,x_2,x_3,\tau_1,\tau_2,\tau_3;t,\tau)\equiv\sup_{T\times T}\{d_0^{-1}\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j\left|\Omega_{is_j}(x_1+2\tau_1\sqrt{\mu\tau},x_2+2\tau_2\sqrt{\mu\tau},x_3+2\tau_3\times\right. \\ \left.\times\sqrt{\mu\tau};t-\tau)\right|)+\sum_{j=1}^{3}\left|\Omega_j(x_1+2\tau_1\sqrt{\mu\tau},x_2+2\tau_2\sqrt{\mu\tau},x_3+2\tau_3\sqrt{\mu\tau};t-\tau)\right|+ \\ +d_0^{-1}[\frac{1}{4\pi}\int_{R^3}(\sum_{i=1}^{3}\frac{\left|\overline{\tau}_i\right|}{\sqrt{(\overline{\tau}_1^2+\overline{\tau}_2^2+\overline{\tau}_3^2)^3}}\{\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j\left|\Omega_{il_j}(x_1+2\tau_1\sqrt{\mu\tau}+\overline{\tau}_1,x_2+2\tau_2\sqrt{\mu\tau}+\overline{\tau}_2,x_3+\right. \end{cases}$$

$$\begin{cases} +2\tau_3\sqrt{\mu\tau}+\bar{\tau}_3;t-\tau\,)\big|)+\sum_{i=1}^{3}(\sum_{j=1}^{3}\big|\Omega_{jl_i}(x_1+2\tau_1\sqrt{\mu\tau}+\bar{\tau}_1,x_2+2\tau_2\sqrt{\mu\tau}+\bar{\tau}_2,x_3+2\tau_3\sqrt{\mu\tau}+ \\ +\bar{\tau}_3;t-\tau\,)\big|)\lambda_i\})d\bar{\tau}_1 d\bar{\tau}_2 d\bar{\tau}_3]\}\le\beta_2\mu, \\ k_0=\dfrac{1}{\sqrt{\pi^3}}\sup_T\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Pi(x_1,x_2,x_3,\tau_1,\tau_2,\tau_3;t,\tau)d\tau_1 d\tau_2 d\tau_3 d\tau\le\beta_2\mu T_0, \\ k_i=\dfrac{1}{\sqrt{\pi^3}}\sup_T\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Pi(x_1,x_2,x_3,\tau_1,\tau_2,\tau_3;t,\tau)\dfrac{|\tau_i|}{\sqrt{\mu\tau}}d\tau_1 d\tau_2 d\tau_3 d\tau\le \\ \le\sqrt{2T_0}\beta_2\sqrt{\mu},(i=\overline{1,3});\quad \beta=\max(\beta_2 T_0;\ 3\sqrt{2T_0}\beta_2), \end{cases} \tag{6.8}$$

and if operators: $\Psi_i,(i=\overline{0,3})$ compressing with a compression factor k_i,

$$\begin{cases} \Psi_i,(i=\overline{0,3}):\ k_i\le\dfrac{h}{4},(h<1),\ (i=\overline{1,3}), \\ \sum_{i=0}^{3}k_i\le\sqrt{\mu}(\beta_2 T_0\sqrt{\mu}+3\sqrt{2T_0}\beta_2)\le\sqrt{\mu}(\sqrt{\mu}+1)\beta=h<1, \\ S_{r_1}(0)=\{Z,W_i:|Z|;\ |W_i|\le r_1,\forall(x_1,x_2,x_3,t)\in T\},(i=\overline{1,3}), \end{cases} \tag{6.9}$$

and

$$\begin{cases} \|\Psi_i[0,0,0,0]\|_C\le r_1(1-h): \\ \|\Psi_i[Z,W_1,W_2,W_3]\|_C\le\|\Psi_i[Z,W_1,W_2,W_3]-\Psi_i[0,0,0,0]\|_C+\|\Psi_i[0,0,0,0]\|_C\le \\ \le k_i 4r_1+r_1(1-h)\le hr_1+r_1(1-h)=r_1;\quad \Psi_i:S_{r_1}(0)\to S_{r_1}(0),\ (i=\overline{0,3}). \end{cases} \tag{6.10}$$

Then on the basis of contraction mapping principle system (6.7) is solvable and solution of this system we can find on the basis of Picard’s method

$$\begin{cases} Z_{n+1}=\Psi_0[Z_n,W_{1,n},W_{2,n},W_{3,n}], \\ W_{i,n+1}=\Psi_i[Z_n,W_{1,n},W_{2,n},W_{3,n}],\ (n=0,1,\ldots;\ \ Z_0=0;\ \ W_{i,0}=0;\ i=\overline{1,3}), \\ E_{n+1}=\|Z_{n+1}-Z_n\|_C+\sum_{i=1}^{3}\|W_{i,n+1}-W_{i,n}\|_C;\ E_n=\|Z_n-Z_{n-1}\|_C+\sum_{i=1}^{3}\|W_{i,n}-W_{i,n-1}\|_C: \\ \|Z_{n+1}-Z_n\|_C\le k_0E_n;\quad \|W_{i,n+1}-W_{i,n}\|_C\le k_iE_n,(i=\overline{1,3}), \\ E_{n+1}\le hE_n\le\ldots\le h^nE_1\xrightarrow[n\to\infty]{h<1}0, \\ \|Z_{n+k}-Z_n\|_C\le\sum_{j=0}^{k-1}k_0E_{n+j};\quad \|W_{i,n+k}-W_{i,n}\|_C\le\sum_{j=0}^{k-1}k_iE_{n+j},(i=\overline{1,3}), \\ E_{n+k}\le h\sum_{j=0}^{k-1}E_{n+j}\le\ldots\le h\sum_{j=0}^{k-1}h^{n+j-1}E_1\le E_1h^n\sum_{j=0}^{k-1}h^j\le E_1h^n\dfrac{1}{1-h}\xrightarrow[n\to\infty]{h<1}0, \\ U_0=\|Z\|_C+\sum_{i=1}^{3}\|W_i\|_C;\ U_{n+1}\equiv\|Z_{n+1}-Z\|_C+\sum_{i=1}^{3}\|W_{i,n+1}-W_i\|_C, \\ U_{n+1}\le h^{n+1}U_0\xrightarrow[n\to\infty]{h<1}0, \\ Z_{n+1}\xrightarrow[n\to\infty]{h<1}Z\equiv H;\quad W_{i,n+1}\xrightarrow[n\to\infty]{h<1}W_i,\forall(x_1,x_2,x_3,t)\in T,(i=\overline{1,3}). \end{cases} \tag{6.11}$$

Hence on the basis of (6.2) and

$$\upsilon_{i,n+1} = \lambda_i[\vartheta_0(x_1,x_2,x_3) + Z_{n+1}(x_1,x_2,x_3,t)] + \Omega_i(x_1,x_2,x_3,t), (n = 0,1,2,...; i = \overline{1,3}), \quad (6.12)$$

we will receive

$$\left\|\upsilon_{i,n+1} - \upsilon_i\right\|_C \le \lambda_i \left\|Z_{n+1} - Z\right\|_C \le \lambda_i h U_n \le \lambda_i h^{n+1} U_0 \xrightarrow[n\to\infty]{h<1} 0, (i = \overline{1,3}). \quad (6.13)$$

And it means that sequence $\{\upsilon_{i,n}\}_0^\infty$ converging to a limit $\tilde{C}^{3,1}(T) \ni \upsilon_i, (i = \overline{1,3}):$

$$\upsilon_{i,n+1} \xrightarrow[n\to\infty]{h<1} \upsilon_i, \forall(x_1,x_2,x_3,t) \in T, (i = \overline{1,3}), \quad (6.14)$$

i.e. under conditions (1.2), (6.1), (6.2), (6.8) - (6.10) and (6.14) problem Navier-Stokes has the smooth single solution in $\tilde{C}^{3,1}_{n=3}(T)$ in a kind (6.2).

6.2. Fluid with Viscosity $1 < \mu_0 = \mu < \infty$, when $\mathrm{div} f = 0$

In this paragraph we will consider a liquid with viscosity and with a Reynolds small number of where all inertial participants contain in the equations Navier-Stokes. Thus we will consider, methods of integrated transformations on the basis of integrals of Poisson's type, when $1 < \mu_0 = \mu < \infty$.

The decision method, from where follows of equations integration of Navier-Stokes in a case

$$\begin{cases} \upsilon_i\big|_{t=0} = \upsilon_{i0}(x_1,x_2,x_3) \equiv \lambda_i \vartheta_0(x_1,x_2,x_3), (0 < \lambda_i = \mathrm{const}; i = \overline{1,3}), \\ \sum_{i=1}^{3} \lambda_i \vartheta_{0x_i} = 0,\ (i = \overline{1,3}),\ \Delta\vartheta_0 = 0;\ \vartheta_0 \in R^3;\ \mathrm{div} f = 0;\ f = (f_1, f_2, f_3), \\ \sup_T \left|D^k f_i\right| \le N_0 = \mathrm{const}, \forall(x_1,x_2,x_3,t) \in T;\ r = \sqrt{(x_1 - s_1)^2 + (x_2 - s_2)^2 + (x_3 - s_3)^2}, \\ K_i(x_1,x_2,x_3,t) \equiv \frac{1}{\sqrt{\mu}} \frac{1}{8\sqrt{\pi^3}} \int_0^t \int_{R^3} \exp(-\frac{r^2}{4\mu(t-s)}) \frac{1}{\sqrt{(\mu(t-s))^3}} f_i(s_1,s_2,s_3,s) ds_1 ds_2 ds_3 ds = \\ = \frac{1}{\sqrt{\mu}} \frac{1}{\sqrt{\pi^3}} \int_0^t \int_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2)) f_i(x_1 + 2\tau_1\sqrt{\mu(t-s)}, x_2 + 2\tau_2\sqrt{\mu(t-s)}, x_3 + \\ +2\tau_3\sqrt{\mu(t-s)}; s) d\tau_1 d\tau_2 d\tau_3 ds, (s_i - x_i = 2\tau_i\sqrt{\mu(t-s)};\ i = \overline{1,3}), \end{cases} \quad (6.15)$$

is a major factor of this point.

Therefore we enter for definition a component of speeds:

$$\begin{cases} \upsilon_i = \lambda_i[\vartheta_0(x_1,x_2,x_3) + Z(x_1,x_2,x_3,t)] + K_i(x_1,x_2,x_3,t), \forall(x_1,x_2,x_3,t) \in T, (i = \overline{1,3}), \\ Z\big|_{t=0} = 0, \forall(x_1,x_2,x_3) \in R^3, \\ \mathrm{div}\, v = 0: \\ \sum_{i=1}^{3} \lambda_i Z_{x_i} = 0;\ \sum_{i=1}^{3} \lambda_i \vartheta_{0x_i} = 0;\ \sum_{i=1}^{3} K_{ix_i} = 0, \end{cases} \quad (6.16)$$

at that

$$
\begin{cases}
\operatorname{div} v = 0 : \sum_{i=1}^{3} \lambda_i Z_{x_i} = 0; \quad \sum_{i=1}^{3} \lambda_i \vartheta_{0x_i} = 0; \quad \sum_{i=1}^{3} K_{ix_i} = 0, \\
K_{it} \equiv \frac{1}{\sqrt{\mu}} f_i(x_1, x_2, x_3, t) + \frac{1}{\sqrt{\mu}} \frac{1}{\sqrt{\pi^3}} \int_0^t \int_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2))[\sum_{j=1}^{3} \frac{\tau_j \sqrt{\mu}}{\sqrt{t-s}} f_{il_j}(x_1 + \\
+2\tau_1 \sqrt{\mu(t-s)}, x_2 + 2\tau_2 \sqrt{\mu(t-s)}, x_3 + 2\tau_3 \sqrt{\mu(t-s)}; s)] d\tau_1 d\tau_2 d\tau_3 ds, (i = \overline{1,3}), \\
K_{ix_j} = \frac{1}{\sqrt{\mu}} \frac{1}{8\sqrt{\pi^3}} \int_0^t \int_{R^3} \frac{-(x_j - s_j)}{2\mu(t-s)} \frac{1}{(\sqrt{\mu(t-s)})^3} \exp(-\frac{r^2}{4\mu(t-s)}) f_i(s_1, s_2, s_3, s) \times \\
\times ds_1 ds_2 ds_3 ds = \frac{1}{\sqrt{\mu}} \frac{1}{\sqrt{\pi}^3} \int_0^t \int_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2)) \frac{\tau_j}{\sqrt{\mu(t-s)}} f_i(x_1 + 2\tau_1 \sqrt{\mu(t-s)}, x_2 + \\
+2\tau_2 \sqrt{\mu(t-s)}, x_3 + 2\tau_3 \sqrt{\mu(t-s)}; s) d\tau_1 d\tau_2 d\tau_3 ds, (i = \overline{1,3};\ j = \overline{1,3}), \\
K_{ix_j^2} = \frac{1}{\sqrt{\mu}} \frac{1}{\sqrt{\pi}^3} \int_0^t \int_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2)) \frac{\tau_j}{\sqrt{\mu(t-s)}} f_{il_j}(x_1 + 2\tau_1 \sqrt{\mu(t-s)}, x_2 + \\
+2\tau_2 \sqrt{\mu(t-s)}, x_3 + 2\tau_3 \sqrt{\mu(t-s)}; s) d\tau_1 d\tau_2 d\tau_3 ds, \\
\mu \Delta K_i = \frac{1}{\sqrt{\mu}} \frac{1}{\sqrt{\pi^3}} \int_0^t \int_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2))[\sum_{j=1}^{3} \frac{\tau_j \sqrt{\mu}}{\sqrt{t-s}} f_{il_j}(x_1 + 2\tau_1 \sqrt{\mu(t-s)}, x_2 + \\
+2\tau_2 \sqrt{\mu(t-s)}, x_3 + 2\tau_3 \sqrt{\mu(t-s)}; s)] d\tau_1 d\tau_2 d\tau_3 ds,\ (l_j = x_j + 2\tau_j \sqrt{\mu(t-s)};\ i, j = \overline{1,3}), \\
\sum_{j=1}^{3} \upsilon_j \upsilon_{ix_j} \equiv \sum_{j=1}^{3} \lambda_j (\vartheta_0 + Z) K_{ix_j} + \sum_{j=1}^{3} \lambda_i (\vartheta_{0x_j} + Z_{x_j}) K_j + \sum_{j=1}^{3} K_j K_{ix_j}, \\
\sum_{j=1}^{3} \lambda_j (\vartheta_0 + Z) \lambda_i (\vartheta_{0x_j} + Z_{x_j}) = \lambda_i (\vartheta_0 + Z) \sum_{j=1}^{3} \lambda_j (\vartheta_{0x_j} + Z_{x_j}) = 0, \\
\upsilon_{it} \equiv \lambda_i Z_t + K_{it},\ \ (i = \overline{1,3}), \\
\mu \Delta \upsilon_i \equiv \mu[\lambda_i \Delta Z + \Delta K_i],\ (i = \overline{1,3}), \\
\upsilon_{it} - \mu \Delta \upsilon_i \equiv \lambda_i Z_t + \frac{1}{\sqrt{\mu}} f_i(x_1, x_2, x_3, t) + \frac{1}{\sqrt{\mu}} \frac{1}{\sqrt{\pi^3}} \int_0^t \int_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2))[\sum_{j=1}^{3} \frac{\tau_j \sqrt{\mu}}{\sqrt{t-s}} \times \\
\times f_{il_j}(x_1 + 2\tau_1 \sqrt{\mu(t-s)}, x_2 + 2\tau_2 \sqrt{\mu(t-s)}, x_3 + 2\tau_3 \sqrt{\mu(t-s)}; s)] d\tau_1 d\tau_2 d\tau_3 ds - \\
- \frac{1}{\sqrt{\mu}} \frac{1}{\sqrt{\pi^3}} \int_0^t \int_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2))[\sum_{j=1}^{3} \frac{\tau_j \sqrt{\mu}}{\sqrt{t-s}} f_{il_j}(x_1 + 2\tau_1 \sqrt{\mu(t-s)}, x_2 + 2\tau_2 \sqrt{\mu(t-s)}, \\
x_3 + 2\tau_3 \sqrt{\mu(t-s)}; s)] d\tau_1 d\tau_2 d\tau_3 ds - \mu \lambda_i \Delta Z = \lambda_i Z_t + \frac{1}{\sqrt{\mu}} f_i - \mu \lambda_i \Delta Z,\ \ (i = \overline{1,3}).
\end{cases}
\tag{6.17}
$$

Hence on the basis of (6.15)-(6.17) we will receive

$$
\lambda_i Z_t + \sum_{j=1}^{3} \lambda_j (\vartheta_0 + Z) K_{ix_j} + \sum_{j=1}^{3} K_j \lambda_i (\vartheta_{0x_j} + Z_{x_j}) + \sum_{j=1}^{3} K_j K_{ix_j} = (1 - \frac{1}{\sqrt{\mu}}) f_i - \frac{1}{\rho} P_{x_i} +
\tag{6.18}
$$

$$
+ \mu \lambda_i \Delta Z, (i = \overline{1,3}).
$$

Then for incompressible currents with a friction the equations of Navier-Stokes (1.1) become simpler as take place (6.15), (6.17). Therefore a system (1.1) with account [(6.5) - (6.7), here instead of Ω_i we will consider K_i] we will receive

$$\left\{\begin{array}{l} Z_{x_i} = W_i, \forall (x_1,x_2,x_3,t) \in T, (i = \overline{1,3}), \\ Z = M_1 + \frac{1}{8\sqrt{\pi^3}} \int_0^t \int_{R^3} \exp(-\frac{r^2}{4\mu(t-s)}) \frac{1}{(\sqrt{\mu(t-s)})^3} (Q[Z,W_1,W_2,W_3])(s_1,s_2,s_3,s) \times \\ \times ds_1 ds_2 ds_3 ds = M_1 + \frac{1}{\sqrt{\pi^3}} \int_0^t \int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2))(Q[Z,W_1,W_2,W_3])(x_1 + 2\tau_1\sqrt{\mu(t-s)}, \\ x_2 + 2\tau_2\sqrt{\mu(t-s)}, x_3 + 2\tau_3\sqrt{\mu(t-s)}; s) d\tau_1 d\tau_2 d\tau_3 ds \equiv (\overline{\Psi}_0[Z,W_1,W_2,W_3])(x_1,x_2,x_3,t), \\ W_i = M_{1x_i} + \frac{1}{8\sqrt{\pi^3}} \int_0^t \int_{R^3} (\exp(-\frac{r^2}{4\mu(t-s)})) \frac{-(x_i - s_i)}{2\mu(t-s)} \frac{1}{(\sqrt{\mu(t-s)})^3} (Q[Z,W_1,W_2,W_3]) \times \\ \times (s_1,s_2,s_3,s) ds_1 ds_2 ds_3 ds = M_{1x_i} + \frac{1}{\sqrt{\pi^3}} \int_0^t \int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)) \frac{\tau_i}{\sqrt{\mu(t-s)}} \times \\ \times (Q[Z,W_1,W_2,W_3])(x_1 + 2\tau_1\sqrt{\mu(t-s)}, x_2 + 2\tau_2\sqrt{\mu(t-s)}, x_3 + 2\tau_3\sqrt{\mu(t-s)}; s) \times \\ \times d\tau_1 d\tau_2 d\tau_3 ds \equiv (\overline{\Psi}_i[Z,W_1,W_2,W_3])(x_1,x_2,x_3,t), \end{array}\right. \qquad (6.19)$$

where

$$\left\{\begin{array}{l} M_1(x_1,x_2,x_3,t) \equiv \frac{1}{\sqrt{\pi^3}} \int_0^t \int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)) \Phi_0(x_1 + 2\tau_1\sqrt{\mu(t-s)}, x_2 + 2\tau_2 \times \\ \times \sqrt{\mu(t-s)}, x_3 + 2\tau_3\sqrt{\mu(t-s)}; s) d\tau_1 d\tau_2 d\tau_3 ds, (s_i - x_i = 2\tau_i\sqrt{\mu(t-s)};\ i = \overline{1,3}), \\ \Delta \frac{1}{\rho} P = -\{F_0 + \tilde{B}_*[Z_{x_1}, Z_{x_2}, Z_{x_3}]\}, \\ (\tilde{B}_*[Z_{x_1}, Z_{x_2}, Z_{x_3}])(x_1,x_2,x_3,t) \equiv \sum_{i=1}^{3} (\sum_{j=1}^{3} \lambda_j K_{ix_j}) Z_{x_i} + \sum_{i=1}^{3} (\sum_{j=1}^{3} K_{jx_i} Z_{x_j}) \lambda_i, \\ \mathrm{div} f = 0;\ F_0(x_1,x_2,x_3,t) \equiv \sum_{i=1}^{3} \frac{\partial}{\partial x_i} (\sum_{j=1}^{3} K_j K_{ix_j}) + \sum_{i=1}^{3} \lambda_i (\sum_{j=1}^{3} K_{jx_i} \vartheta_{0x_j}) + \sum_{i=1}^{3} \vartheta_{0x_i} (\sum_{j=1}^{3} \lambda_j K_{ix_j}), \\ \frac{1}{\rho} P = \frac{1}{4\pi} \int_{R^3} \frac{1}{r} \{F_0(s_1,s_2,s_3,t) + (\tilde{B}_*[Z_{s_1}, Z_{s_2}, Z_{s_3}])(s_1,s_2,s_3,t)\} ds_1 ds_2 ds_3, \\ \frac{1}{\rho} P_{x_i} = \frac{1}{4\pi} \int_{R^3} \frac{\tau_i}{\sqrt{(\tau_1^2+\tau_2^2+\tau_3^2)^3}} \{F_0(x_1+\tau_1, x_2+\tau_2, x_3+\tau_3; t) + (\tilde{B}_*[Z_{h_1}, Z_{h_2}, Z_{h_3}])(x_1 + \\ +\tau_1, x_2+\tau_2, x_3+\tau_3; t)\} d\tau_1 d\tau_2 d\tau_3, \quad (s_i - x_i = \tau_i; h_i = x_i + \tau_i; i = \overline{1,3}), \\ Z_t + d_0^{-1} \sum_{i=1}^{3} [\sum_{j=1}^{3} \lambda_j (\vartheta_0 + Z) K_{ix_j} + \sum_{j=1}^{3} K_j K_{ix_j}] + \sum_{j=1}^{3} K_j (\vartheta_{0x_j} + Z_{x_j}) = d_0^{-1} \{\sum_{i=1}^{3} [(1 - \frac{1}{\sqrt{\mu}}) f_i - \\ - \frac{1}{4\pi} \int_{R^3} \frac{\tau_i}{\sqrt{(\tau_1^2+\tau_2^2+\tau_3^2)^3}} \{F_0(x_1+\tau_1, x_2+\tau_2, x_3+\tau_3; t) + (\tilde{B}_*[Z_{h_1}, Z_{h_2}, Z_{h_3}])(x_1 + \end{array}\right.$$

$$\begin{cases} +\tau_1, x_2+\tau_2, x_3+\tau_3; t)\}d\tau_1 d\tau_2 d\tau_3]\} + \mu \Delta Z, \\ (Q[Z, Z_{s_1}, Z_{s_2}, Z_{s_3}])(s_1, s_2, s_3, s) \equiv -\{d_0^{-1}[Z(s_1, s_2, s_3, s)\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j K_{is_j}(s_1, s_2, s_3, s))] + \\ +\sum_{j=1}^{3} Z_{s_j}(s_1, s_2, s_3, s)K_j(s_1, s_2, s_3, s) + d_0^{-1}[\frac{1}{4\pi}\int_{R^3}(\sum_{i=1}^{3}\frac{\overline{\tau}_i}{\sqrt{(\overline{\tau}_1^2+\overline{\tau}_2^2+\overline{\tau}_3^2)^3}}\{(\tilde{B}_*[Z_{\overline{h}_1}, Z_{\overline{h}_2}, Z_{\overline{h}_3}])(s_1 + \\ +\overline{\tau}_1, s_2+\overline{\tau}_2, s_3+\overline{\tau}_3; s)\})d\overline{\tau}_1 d\overline{\tau}_2 d\overline{\tau}_3]\}, \quad (\overline{h}_i = s_i + \overline{\tau}_i; \quad i = \overline{1,3}), \\ \Phi_0(x_1, x_2, x_3, t) \equiv -\sum_{j=1}^{3} K_j \vartheta_{0x_j} + d_0^{-1}[\sum_{i=1}^{3}(1-\frac{1}{\sqrt{\mu}})f_i - \sum_{i=1}^{3}\vartheta_0(\sum_{j=1}^{3}\lambda_j K_{ix_j}) - \sum_{i=1}^{3}(\sum_{j=1}^{3}K_j K_{ix_j}) - \\ -\frac{1}{4\pi}\int_{R^3}\{\sum_{i=1}^{3}\frac{\tau_i}{\sqrt{(\tau_1^2+\tau_2^2+\tau_3^2)^3}}(F_0(x_1+\tau_1, x_2+\tau_2, x_3+\tau_3; t))\}d\tau_1 d\tau_2 d\tau_3]; \; d_0 = \sum_{i=1}^{3}\lambda_i > 0. \end{cases}$$

If takes place:

$$\begin{cases} \forall (x_1, x_2, x_3, t) \in T; M_1, Y, K_i: \\ \sup_T \left|D^k M_1(x_1, x_2, x_3, t)\right| \le \beta_1, \; (k = \overline{0,3}; \; t - s = \tau), \\ \sup_{T\times T} Y(x_1, x_2, x_3, \tau_1, \tau_2, \tau_3; t, \tau) \equiv \sup_{T\times T}\{d_0^{-1}\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j \left|K_{is_j}(x_1 + 2\tau_1\sqrt{\mu\tau}, x_2 + 2\tau_2\sqrt{\mu\tau}, x_3 + \right. \\ \left. +2\tau_3\sqrt{\mu\tau}; t-\tau)\right|) + \sum_{j=1}^{3}\left|K_j(x_1+2\tau_1\sqrt{\mu\tau}, x_2+2\tau_2\sqrt{\mu\tau}, x_3+2\tau_3\sqrt{\mu\tau}; t-\tau)\right| + \\ +d_0^{-1}[\frac{1}{4\pi}\int_{R^3}(\sum_{i=1}^{3}\frac{\left|\overline{\tau}_i\right|}{\sqrt{(\overline{\tau}_1^2+\overline{\tau}_2^2+\overline{\tau}_3^2)^3}}\{\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j\left|K_{il_j}(x_1+2\tau_1\sqrt{\mu\tau}+\overline{\tau}_1, x_2+2\tau_2\sqrt{\mu\tau}+\overline{\tau}_2, x_3 + \right. \\ \left. +2\tau_3\sqrt{\mu\tau}+\overline{\tau}_3; t-\tau)\right|) + \sum_{i=1}^{3}(\sum_{j=1}^{3}\left|K_{jl_i}(x_1+2\tau_1\sqrt{\mu\tau}+\overline{\tau}_1, x_2+2\tau_2\sqrt{\mu\tau}+\overline{\tau}_2, x_3+2\tau_3\sqrt{\mu\tau} + \right. \\ \left. +\overline{\tau}_3; t-\tau)\right|)\lambda_i\})d\overline{\tau}_1 d\overline{\tau}_2 d\overline{\tau}_3]\} \le \frac{1}{\sqrt{\mu}}\beta_2, \\ k_0 = \frac{1}{\sqrt{\pi^3}}\sup_T\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))Y(x_1, x_2, x_3, \tau_1, \tau_2, \tau_3; t, \tau)d\tau_1 d\tau_2 d\tau_3 d\tau \le \frac{1}{\sqrt{\mu}}\beta_2 T_0, \\ k_i = \frac{1}{\sqrt{\pi^3}}\sup_T\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))Y(x_1, x_2, x_3, \tau_1, \tau_2, \tau_3; t, \tau)\frac{\left|\tau_i\right|}{\sqrt{\mu\tau}}d\tau_1 d\tau_2 d\tau_3 d\tau \le \\ \le \sqrt{2T_0}\beta_2\frac{1}{\mu}, (i = \overline{1,3}), \quad \beta_* = \max(\beta_2 T_0; 3\sqrt{2T_0}\beta_2), \end{cases} \tag{6.20}$$

and

$$\begin{cases} \overline{\Psi}_i, (i = \overline{0,3}): \; k_i \le \frac{h}{4}, (h < 1), (i = \overline{1,3}), \\ \sum_{i=0}^{3} k_i \le \frac{1}{\sqrt{\mu}}(\beta_2 T_0 + 3\sqrt{2T_0}\beta_2\frac{1}{\sqrt{\mu}}) \le \frac{1}{\sqrt{\mu}}(1+\frac{1}{\sqrt{\mu}})\beta_* = h < 1, (1 < \mu = \mu_0 < \infty), \end{cases}$$

$$\begin{cases} S_{r_1}(0) = \{Z, W_i : |Z|;\ |W_i| \le r_1, \forall (x_1, x_2, x_3, t) \in T\}, (i = \overline{1,3}), \\ \|\overline{\Psi}_i[0,0,0,0]\|_C \le r_1(1-h): \\ \|\overline{\Psi}_i[Z, W_1, W_2, W_3]\|_C \le \|\overline{\Psi}_i[Z, W_1, W_2, W_3] - \overline{\Psi}_i[0,0,0,0]\|_C + \|\overline{\Psi}_i[0,0,0,0]\|_C \le \\ \le k_i 4 r_1 + r_1(1-h) \le h r_1 + r_1(1-h) = r_1, \\ \overline{\Psi}_i : S_{r_1}(0) \to S_{r_1}(0),\ (i = \overline{0,3}). \end{cases} \tag{6.21}$$

The solution of this system we can find on the basis of Picard's method (6.11). Then considering results (6.12)-(6.14), hence we will receive, that sequence $\{\upsilon_{i,n}\}_0^\infty$ converging to a limit $\tilde{C}^{3,1}(T) \ni \upsilon_i, (i = \overline{1,3}):$

$$\upsilon_{i,n+1} \xrightarrow[n \to \infty]{h<1} \upsilon_i \equiv H_i, \forall (x_1, x_2, x_3, t) \in T, (i = \overline{1,3}). \tag{6.22}$$

So as consequence of paragraphs 6.1 and 6.2 we will receive following statements:

Theorem 7. The Navier-Stoke's nonstationary problem (1.1)-(1.3) is solvable at $\tilde{C}^{3,1}_{n=3}(T)$, when are fulfilled the conditions:

1) (6.1), (6.2), (6.8) - (6.10), (6.14) and $0 < \mu < 1, \operatorname{div} f = 0,$ or

2) (6.15)-(6.17), (6.20), (6.21), (6.22) and $1 < \mu = \mu_0 < \infty, \operatorname{div} f = 0.$

7. Fluid with Viscosity, when $v \in R^n, (x \in R^n; t \in [0, T_0])$

Here we will show that at certain mathematical transformations of the equation Navier-Stokes led to a linear kind, when $v \in R^n$. So, once again clearly confirmed [12], that is, that the Navier-Stokes equations with viscosity have solutions in analytical form, which is based on the Picard's method.

Offered methods of integrated transformations in paragraph 4 have been based on integrals of Poisson's type. These methods are entered so that to transform nonlinear problems of Navier-Stokes in linear problems of heat conductivity [13]. Our purpose – to apply these methods for the equation Navier-Stokes in a case $v = (\upsilon_1, \ldots, \upsilon_n),$

$$\frac{\partial \upsilon_i}{\partial t} + \sum_{j=1}^{n} \upsilon_j \frac{\partial \upsilon_i}{\partial x_j} = f_i - \frac{1}{\rho} P_{x_i} + \mu \Delta \upsilon_i, (i = \overline{1,n}), \tag*{(1.1)$_n$}$$

$$\operatorname{div} v = 0, \forall (x,t) \in T_* = R^n \times [0, T_0], \tag*{(1.2)$_n$}$$

$$\upsilon_i\big|_{t=0} = \upsilon_{i0}(x_1, x_2, \ldots, x_n), \forall (x_1, x_2, \ldots, x_n) \in R^n, (i = \overline{1,n}). \tag*{(1.3)$_n$}$$

From the received results follows that system Navier-Stokes (1.1)$_n$ in the conditions of (1.2)$_n$, (1.3)$_n$ can have the analytical smooth single solution in $\tilde{C}_n^{3,1}(T_*)$, (or the conditional-smooth single solution in $G_n^1(D_0 = R^n \times (0,T_0))$).

7.1. Fluid with Very Small Viscosity $0 < \mu < 1$, when $\operatorname{div} f \neq 0$

Let υ_{i0} initial components of a vector of speed v at the moment of time $t = 0$ it is set in a kind $(1.3)_n$:

$$\upsilon_i\Big|_{t=0} = \upsilon_{i0}(x_1,x_2,\ldots,x_n) \equiv \lambda_i \vartheta_0(x_1,x_2,\ldots,x_n),(i = \overline{1,n}), \tag{7.1}$$

where $0 < \lambda_i -$ the known constants. Then speed components v are defined by a rule

$$\begin{cases} \upsilon_i = \lambda_i V(x_1,x_2,\ldots,x_n,t),(i = \overline{1,n}), \\ V\Big|_{t=0} = \vartheta_0(x_1,x_2,\ldots,x_n),\ \forall (x_1,x_2,\ldots,x_n) \in R^n, \\ \operatorname{div} f \neq 0;\ \ \operatorname{div} v = 0: \\ \sum\limits_{j=1}^{n} \lambda_j V_{x_j} = 0;\ \ \sum\limits_{j=1}^{n} \upsilon_j \upsilon_{ix_j} = \lambda_i V \sum\limits_{j=1}^{n} \lambda_j V_{x_j} = 0. \end{cases} \tag{7.2}$$

Hence, the system $(1.1)_n$ will be transformed to a kind

$$\lambda_i V_t = f_i - \frac{1}{\rho} P_{x_i} + \mu \lambda_i \Delta V,(i = \overline{1,n}), \tag{7.3}$$

where V new unknown function which defines the decision on problem Navier-Stokes. Here substitution (7.2) it is equivalent will transform system $(1.1)_n$ in the linear nonuniform equation of a kind (7.3).

For this purpose, at first we will define pressure P. Really, considering APS from system (7.3) we will receive:

$$\begin{cases} \sum\limits_{i=1}^{n} \dfrac{\partial}{\partial x_i}(7.3): \quad \dfrac{1}{\rho}\Delta P = -4\pi F_0,(F_0 \equiv -\dfrac{1}{4\pi}\sum\limits_{i=1}^{n} f_{ix_i}(x_1,x_2,\ldots,x_n,t)), \\ \dfrac{1}{\rho} P = \int\limits_{R^n} F_0(s_1,s_2,\ldots,s_n,t)\dfrac{ds_1 ds_2 \ldots ds_n}{r},(r = \sqrt{\sum\limits_{i=1}^{n}(x_i - s_i)^2}), \\ \dfrac{1}{\rho} P_{x_i} = \int\limits_{R^n} \dfrac{\tau_i F_0(x_1+\tau_1,x_2+\tau_2,\ldots,x_n+\tau_n;t)d\tau_1 d\tau_2 \ldots d\tau_n}{\sqrt{(\tau_1^2+\tau_2^2+\ldots+\tau_n^2)^3}},(s_i - x_i = \tau_i; i = \overline{1,n}). \end{cases} \tag{7.4}$$

Therefore

$$\begin{cases} V_t = \Phi_0(x_1,x_2,\ldots,x_n,t) + \mu\Delta V, \\ \sum\limits_{i=1}^{n} \Phi_{0x_i} = 0,\ \forall (x_1,x_2,\ldots,x_n,t) \in T, \\ (\lambda_1)^{-1}(f_1 - \rho^{-1}P_{x_1}) = (\lambda_2)^{-1}(f_2 - \rho^{-1}P_{x_2}) = \ldots = (\lambda_n)^{-1}(f_n - \rho^{-1}P_{x_n}) \equiv \Phi_0, \end{cases} \tag{7.5}$$

i.e. is the system (7.3) is transformed in the linear equations of heat conductivity with a condition of Cauchy in a kind (7.5), and in a class of functions with smooth enough initial data is correctly put [13, 14]. Accordingly there is an the conditional smooth and single solution of a problem Navier-Stokes in $G^1(D_0)$.

Really from system (7.13), follows:

$$V = \frac{1}{2^n(\sqrt{\pi\mu t})^n}\int_{R^n}\exp(-\frac{r^2}{4\mu t})V_0(s_1,s_2,\dots,s_n)ds_1ds_2\dots ds_n + \frac{1}{2^n\sqrt{\pi^n}}\int_0^t\int_{R^n}\exp(-\frac{r^2}{4\mu(t-s)})\times$$

$$\times\frac{1}{\sqrt{(\mu(t-s))^n}}\Phi_0(s_1,s_2,\dots,s_n,s)ds_1ds_2\dots ds_n ds = \frac{1}{\sqrt{\pi^n}}\int_{R^n}\exp(-(\tau_1^2+\tau_2^2+\dots+\tau_n^2))V_0(x_1+$$

$$+2\tau_1\sqrt{\mu t},x_2+2\tau_2\sqrt{\mu t},\dots,x_n+2\tau_n\sqrt{\mu t})d\tau_1 d\tau_2\dots d\tau_n + \frac{1}{\sqrt{\pi^n}}\int_0^t\int_{R^n}\exp(-(\tau_1^2+\tau_2^2\dots+\tau_n^2))\Phi_0(x_1+ \quad (7.6)$$

$$+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},\dots,x_n+2\tau_n\sqrt{\mu(t-s)};s)d\tau_1 d\tau_2\dots d\tau_n ds \equiv H_0(x_1,x_2,\dots,x_n,t),$$

$$s_i - x_i = 2\tau_i\sqrt{\mu t};\ s_i - x_i = 2\tau_i\sqrt{\mu(t-s)},(i=\overline{1,n}),$$

H_0 – is known function. The limiting case in $G^1(D_0)$, when the solution (7.5) is representing in the form (7.6), when

$$\begin{cases}
\forall(x_1,x_2,\dots,x_n,t)\in T_*;V_0;\Phi_0: \\
\sup_{R^n}\left|D^k V_0\right| \le \beta_1,\ (k=\overline{0,3}),\ \sup_{T_*}\left|D^k\Phi_0(x_1,x_2,\dots,x_n,t)\right| \le \gamma_1, \\
\sup_{T_*}\frac{1}{\sqrt{\pi^n}}\int_0^t\int_{R^n}\exp(-(\tau_1^2+\tau_2^2+\dots+\tau_n^2))\left|D^k\Phi_0(l_1,l_2,\dots,l_n;s)\right|d\tau_1 d\tau_2\dots d\tau_n ds \le \gamma_1 T_0 = \beta_2, \\
\sup_{T_*}\frac{1}{\sqrt{\pi^n}}\int_0^t\int_{R^n}\exp(-(\tau_1^2+\tau_2^2+\dots+\tau_n^2))\frac{1}{\sqrt{t-s}}\sum_{j=1}^n\left|\tau_j\right|\times\left|\Phi_{0l_j}(l_1,l_2,\dots,l_n;s)\right|d\tau_1 d\tau_2\dots d\tau_n ds \le \\
\le n\gamma_1\sqrt{2T_0} = \beta_3,\ (l_i = x_i + 2\tau_i\sqrt{\mu(t-s)};i=\overline{1,n});\quad \sup_{R^n}\int_0^{T_0}\left|\Phi_0(x_1,x_2,\dots,x_n,s)\right|ds \le \gamma_1 T_0 = \beta_2, \\
\sup_{R^n}\frac{1}{\sqrt{\pi^n}}\int_{R^n}\exp(-(\tau_1^2+\tau_2^2+\dots+\tau_n^2))(\sum_{i=1}^n\left|\tau_j\right|\times\left|V_{0\bar{l}_j}(\bar{l}_1,\bar{l}_2,\dots,\bar{l}_n)\right|)d\tau_1 d\tau_2\dots d\tau_n \le \beta_1\frac{1}{\sqrt{\pi^n}}\times \\
\times\{\sum_{i=1}^n(\int_{R^n}\tau_i^2\exp(-(\tau_1^2+\tau_2^2+\dots+\tau_n^2))d\tau_1 d\tau_2\dots d\tau_n)^{\frac{1}{2}}(\int_{R^n}\exp(-(\tau_1^2+\tau_2^2+\dots+\tau_n^2))\times \\
\times d\tau_1 d\tau_2\dots d\tau_n)^{\frac{1}{2}}\} \le n\beta_1\frac{1}{\sqrt{2}},\ (\bar{l}_i = x_i + 2\tau_i\sqrt{\mu t};i=\overline{1,n}), \\
\beta = \max_{1\le i\le 3}\beta_i;\quad \bar{\beta}_0 = \beta(n\sqrt{2\mu T_0}+1+T_0\sqrt{\mu}).
\end{cases} \quad (7.7)$$

Really, estimating (7.6) in $G^1(D_0)$, we have

$$\begin{cases} \|V\|_{G^1(D_0)} = \|V\|_{C^{3,0}(T_*)} + \|V_t\|_{L^1} \le N_* + \bar{\beta}_0, \\ \|V\|_{C^{3,0}(T_*)} = \sum_{0\le|k|\le n} \|D^k V\|_{C(T_*)} \le N_*, \\ \|V\|_{C(T_*)} \le 2\beta, \\ \|V_t\|_{L^1} = \sup_{R^n} \int_0^{T_0} |V_t(x_1,x_2,\dots,x_n,t)|dt \le \beta(n\sqrt{2\mu T_0} + 1 + T_0\sqrt{\mu}) = \bar{\beta}_0. \end{cases} \tag{7.8}$$

Singleness is obvious, as a method by contradiction from (7.6) in $G^1(D_0)$. Results (7.6) with a condition (7.2), (7.7) are received where smoothness of functions is required only on x_i as the derivative of 1st order is in time has $t > 0$. Hence, on a basis transformation (7.2) we will receive decisions of system $(1.1)_n$, which satisfies a condition $(1.2)_n$, i.e.

$$\begin{cases} \upsilon_i = \lambda_i H_0(x_1,x_2,\dots,x_n,t),(i=\overline{1,n}), \\ \sum_{i=1}^n \upsilon_{ix_i} = \sum_{i=1}^n \lambda_i H_{0x_i} = 0, \\ \sum_{i=1}^n \lambda_i H_{0x_i} \equiv \frac{1}{\sqrt{\pi^n}} \int_{R^n} \exp(-(\tau_1^2+\tau_2^2+\dots+\tau_n^2)) \sum_{i=1}^n \lambda_i V_{0h_i}(x_1+2\tau_1\sqrt{\mu t}, x_2+2\tau_2\sqrt{\mu t},\dots,x_n+ \\ +2\tau_n\sqrt{\mu t})d\tau_1 d\tau_2\dots d\tau_n + \frac{1}{\sqrt{\pi^n}} \int_0^t \int_{R^n} \exp(-(\tau_1^2+\tau_2^2+\dots+\tau_n^2)) \sum_{i=1}^n \lambda_i \Phi_{0l_i}(x_1+2\tau_1\sqrt{\mu(t-s)}, x_2+ \\ +2\tau_2\sqrt{\mu(t-s)},\dots,x_n+2\tau_n\sqrt{\mu(t-s)};s)d\tau_1 d\tau_2\dots d\tau_n ds = 0, \\ (h_i = x_i + 2\tau_i\sqrt{\mu t};\ l_i = x_i + 2\tau_i\sqrt{\mu(t-s)};i=\overline{1,n}). \end{cases} \tag{7.9}$$

In the conclusion estimating (7.9) it is had

$$\begin{cases} v = (\upsilon_1,\upsilon_2,\dots,\upsilon_n); \quad \upsilon_i = \lambda_i V,(i=\overline{1,n}): \\ \|v\|_{G^1_{n=3}(D_0)} = \sum_{i=1}^n [\|\lambda_i V\|_{C^{3,0}(T_*)} + \|\lambda_i V_t\|_{L^1}] \le d_0[N_* + \bar{\beta}_0] = d_0 M_0, \\ d_0 = \sum_{i=1}^n \lambda_i; \quad M_0 = N_* + \bar{\beta}_0, \\ \|V\|_{G^1(D_0)} = \|V\|_{C^{3,0}(T_*)} + \|V_t\|_{L^1} \le N_* + \bar{\beta}_0. \end{cases} \tag{7.10}$$

Theorem 8. In the conditions of $(1.2)_n$, (7.1), (7.7) and (7.10) the problem $(1.1)_n$, $(1.2)_n$, (7.1) has a single solution in $G^1_n(D_0)$, which is defined by a rule (7.9).

Remark 5. Alternatively, we can consider, e.g., a class of suitable solutions constructed in $W^2_{n,\lambda}(D_0)$ – weight space of Sobolev's type. So as $\upsilon_{i0} \in C^3(R^n)$, that decision (7.6) of problem Navier-Stokes $(1.1)_n$ - $(1.3)_n$ belongs in $W^2_{n,\lambda}(D_0)$:

$$\begin{cases} W^2_{n,\lambda}(D_0): \quad \|\nu\|_{W^2_{n,\lambda}} = \sum_{i=1}^{n} \|\upsilon_i\|_{\tilde W^2_{(\upsilon_i,\lambda)}}, \\ \tilde W^2_{(\upsilon_i,\lambda)} = \{(x_1,x_2,\dots,x_n,t)\in D_0 : D^k\upsilon_i \in L^2;\ \ \upsilon_{it}\in L^2_\lambda\}, \quad (i=\overline{1,n}), \\ \|\upsilon_i\|_{\tilde W^2_{(\upsilon_i,\lambda)}} = \{\sum_{0\le|k|\le 3} \sup_{R^n}\int_0^{T_0}[D^k\upsilon_i(x_1,x_2,\dots,x_n,t)]^2dt + \sup_{R^n}\int_0^{T_0}\lambda(t)|\upsilon_{it}(x_1,x_2,\dots,x_n,t)|^2dt\}^{\frac{1}{2}}, \end{cases}$$

if takes place

$$\begin{cases} \forall(x_1,x_2,\dots,x_n,t)\in T_*: \\ \sup_{T_*}|D^k\Phi_i| \le \gamma_1, \quad (i=\overline{1,n}; k=\overline{0,3}), \\ 0\le\lambda(t): \int_0^{T_0}\lambda(t)\frac{1}{t}dt = q_0; \quad \int_0^{T_0}\lambda(t)dt = q_1, \\ (\sup_{R^n}\int_0^{T_0}\lambda(s)|\Phi_i(x_1,x_2,\dots,x_n,s)|^2ds)^{\frac{1}{2}} \le \gamma_1\sqrt{q_1} = \beta_4, \\ \beta_* = \max(\beta,\beta_4); \quad \tilde\beta_0 = \beta_*(n\sqrt{\mu q_0}+1+\sqrt{\mu q_1}). \end{cases} \tag{7.11}$$

For this purpose it is enough to show function accessories V in $\tilde W^2_\lambda(D_0)$.

$$\begin{cases} \upsilon_i = \lambda_i V, (i=\overline{1,n}): \quad \tilde W^2_\lambda = \{(x_1,x_2,\dots,x_n,t)\in D_0 : D^kV\in L^2;\ \ V_t\in L^2_\lambda\}, \\ \|V\|_{\tilde W^2_\lambda} = \{\sum_{0\le|k|\le 3}\sup_{R^n}\int_0^{T_0}[D^kV(x_1,x_2,\dots,x_n,t)]^2dt + \sup_{R^n}\int_0^{T_0}\lambda(t)|V_t(x_1,x_2,\dots,x_n,t)|^2dt\}^{\frac{1}{2}}. \end{cases}$$

Let the decision of system (7.5) to represent in the form of (7.6) with conditions (7.7), (7.13). Then estimating (7.6) in $\tilde W^2_\lambda(D_0)$, we have [8]:

$$\begin{cases} \|V\|_{\tilde W^2_{\lambda(D_0)}} \le N_*\sqrt{T_0} + \tilde\beta_0 = M_*, \quad (\beta_*(n\sqrt{\mu q_0}+1+\sqrt{\mu q_1}) = \tilde\beta_0), \\ \|V_t\|_{L^2_\lambda} = (\sup_{R^n}\int_0^{T_0}\lambda(t)|V_t(x_1,x_2,\dots,x_n,t)|^2dt)^{\frac{1}{2}} \le \beta_*(n\sqrt{\mu q_0}+1+\sqrt{\mu q_1}) = \tilde\beta_0, (i=\overline{1,n}). \end{cases} \tag{7.12}$$

Then on a basis (7.2), (7.12) it is had

$$\begin{cases} d_0 = \sum_{i=1}^{n}\lambda_i, \quad (D_0 = R^3\times(0,T_0)), \quad \nu = (\lambda_1 V,\dots,\lambda_n V)): \\ \|\nu\|_{W^2_{n,\lambda}(D_0)} = \sum_{i=1}^{n}\|\upsilon_i\|_{\tilde W^2_{(\upsilon_i,\lambda)}(D_0)} = \sum_{i=1}^{n}\lambda_i\|V\|_{\tilde W^2_\lambda(D_0)} \le d_0(N_*\sqrt{T_0}+\tilde\beta_0) = d_0 M_*, \end{cases}$$

i.e. in the conditions of $(1.2)_n$, $(1.3)_n$, (7.1), (7.7) and (7.11) the problem $(1.1)_n$ - $(1.3)_n$ has a the limited solution in $W^2_{n,\lambda}(D_0)$.

7.2. Fluid with Viscosity $0 < \mu < 1$, when $\mathrm{div} f \neq 0;\ \Delta\vartheta_0 = 0$

I. The overall objective of this point: to change a method (7.2) so that the received analytical decision of a problem Navier-Stokes with viscosity, belonged in $\tilde{C}_n^{3,1}(T_*)$.

If takes place

$$\begin{cases} \upsilon_i\big|_{t=0} = \upsilon_{i0}(x_1,\dots,x_n) \equiv \lambda_i\vartheta_0(x_1,\dots,x_n), i=\overline{1,n}, \\ \sum\limits_{j=1}^{n}\lambda_j\vartheta_{0x_j} = 0;\ \Delta\vartheta_0 = 0;\ \vartheta_0 \in C^3(R^n),\ \mathrm{div} f \neq 0;\ \sup\limits_{T_*}\left|D^k f_i\right| \le N_0,\ (i=\overline{1,n}; k=\overline{0,4}), \end{cases} \tag{7.13}$$

that we will use transformation of a kind

$$\begin{cases} \upsilon_i = \lambda_i[\vartheta_0(x_1,x_2,\dots,x_n) + Z(x_1,x_2,\dots,x_n,t)], \forall(x_1,x_2,\dots,x_n,t)\in T_*, (i=\overline{1,n}), \\ Z\big|_{t=0} = 0,\ \forall(x_1,x_2,\dots,x_n)\in R^n, \\ \mathrm{div}\, v = 0:\ \sum\limits_{j=1}^{n}\lambda_j Z_{x_j} = 0;\ \sum\limits_{j=1}^{n}\lambda_j\vartheta_{0x_j} = 0, \\ \sum\limits_{j=1}^{n}\upsilon_j\upsilon_{ix_j} = \lambda_i\vartheta_0\sum\limits_{j=1}^{n}\lambda_j\vartheta_{0x_j} + \lambda_i\vartheta_0\sum\limits_{j=1}^{n}\lambda_j Z_{x_j} + \lambda_i Z\sum\limits_{j=1}^{n}\lambda_j\vartheta_{0x_j} + \lambda_i Z\sum\limits_{j=1}^{n}\lambda_j Z_{x_j} = 0, \end{cases} \tag{7.14}$$

where $0 < \lambda_i$ – the known constants. Hence, the system $(1.1)_n$ will be transformed to a kind

$$\lambda_i Z_t = f_i - \frac{1}{\rho}P_{x_i} + \mu\lambda_i\Delta Z, (i=\overline{1,n}). \tag{7.15}$$

From system (7.15), considering conditions (7.13), (7.14), and having entered APS we have the equation

$$\begin{cases} \sum\limits_{i=1}^{n}\frac{\partial}{\partial x_i}(7.15):\quad \frac{1}{\rho}\Delta P = -4\pi F_0, \left(F_0 \equiv -\frac{1}{4\pi}\sum\limits_{i=1}^{n} f_{ix_i}(x_1,x_2,\dots,x_n,t)\right), \\ \frac{1}{\rho}P = \int\limits_{R^n} F_0(s_1,s_2,\dots,s_n,t)\frac{ds_1 ds_2\dots ds_n}{r}, \\ \frac{1}{\rho}P_{x_i} = \int\limits_{R^n}\frac{\tau_i F_0(x_1+\tau_1,x_2+\tau_2,\dots,x_n+\tau_n;t)d\tau_1 d\tau_2\dots d\tau_n}{\sqrt{(\tau_1^2+\tau_2^2+\dots+\tau_n^2)^3}}, \\ r = \sqrt{(x_1-s_1)^2+(x_2-s_2)^2+\dots+(x_n-s_n)^2};\quad s_i - x_i = \tau_i,\quad (i=\overline{1,n}). \end{cases} \tag{7.16}$$

Hence the system (7.15) will be transformed to a kind

$$\begin{cases} Z_t = \Phi_0 + \mu\Delta Z,\ \forall(x_1,x_2,\dots,x_n,t)\in T_*, \\ Z\big|_{t=o} = 0, \\ (\lambda_1)^{-1}(f_1 - \rho^{-1}P_{x_1}) = (\lambda_2)^{-1}(f_2 - \rho^{-1}P_{x_2}) = \dots = (\lambda_n)^{-1}(f_n - \rho^{-1}P_{x_n}) \equiv \Phi_0(x_1,x_2,\dots,x_n,t). \end{cases} \tag{7.15*}$$

Then the decision of problem (7.15)* is presented in a kind

$$Z=\frac{1}{2^n\sqrt{\pi^n}}\int_0^t\int_{R^n}\exp(-\frac{r^2}{4\mu(t-s)})\frac{1}{(\sqrt{\mu(t-s)})^n}\Phi_0(s_1,s_2,\ldots,s_n,s)ds_1ds_2\ldots ds_nds=$$

$$=\frac{1}{\sqrt{\pi^n}}\int_0^t\int_{R^n}\exp(-(\tau_1^2+\tau_2^2+\ldots+\tau_n^2))\Phi_0(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},\ldots,x_n+2\tau_n\times \tag{7.17}$$

$$\times\sqrt{\mu(t-s)};s)d\tau_1d\tau_2\ldots d\tau_nds\equiv H,\forall(x_1,x_2,\ldots,x_,t)\in T_*,(s_i-x_i=2\tau_i\sqrt{\mu(t-s)};i=\overline{1,n}).$$

Here H – known function.

The found decision (7.17) satisfies system (7.15)*. Really, having calculated partial derivative of system (7.17):

$$\begin{cases}Z_t=\Phi_0+\frac{1}{\sqrt{\pi^n}}\int_0^t\int_{R^n}\exp(-(\tau_1^2+\tau_2^2+\ldots+\tau_n^2))\sum_{j=1}^n\sqrt{\mu}\frac{\tau_j}{\sqrt{t-s}}\Phi_{0l_j}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+\\+2\tau_2\sqrt{\mu(t-s)},\ldots,x_n+2\tau_n\sqrt{\mu(t-s)};s)d\tau_1d\tau_2\ldots d\tau_nds,\\Z_{x_j}=\frac{1}{\sqrt{\pi^n}}\int_0^t\int_{R^n}\exp(-(\tau_1^2+\tau_2^2+\ldots+\tau_n^2))\Phi_{0l_j}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},\ldots,x_n+\\+2\tau_n\sqrt{\mu(t-s)};s)d\tau_1d\tau_2\ldots d\tau_nds,\\Z_{x_j^2}=\frac{1}{\sqrt{\pi^n}}\int_0^t\int_{R^n}\exp(-(\tau_1^2+\tau_2^2+\ldots+\tau_n^2))\Phi_{0l_j^2}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},\ldots,x_n+\\+2\tau_n\sqrt{\mu(t-s)};s)d\tau_1d\tau_2\ldots d\tau_nds,\ (l_j=x_j+2\tau_j\sqrt{\mu(t-s)};\ j=\overline{1,n}),\end{cases} \tag{7.18}$$

and substituting (7.18) in (7.15)*, we have

$$\begin{cases}Z\big|_{t=0}=0,\ \forall(x_1,x_2,\ldots,x_n)\in R^n;\quad(0,1)\in\mu;\ \forall(x_1,x_2,\ldots,x_n,t)\in T_*:\\0=Z_t-\Phi_0-\mu\Delta Z\equiv\Phi_0+\frac{1}{\sqrt{\pi^n}}\int_0^t\int_{R^n}\exp(-(\tau_1^2+\tau_2^2+\ldots+\tau_n^2))\sum_{j=1}^n\sqrt{\mu}\frac{\tau_j}{\sqrt{t-s}}\Phi_{0l_j}(x_1+\\+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},\ldots,x_n+2\tau_n\sqrt{\mu(t-s)};s)d\tau_1d\tau_2\ldots d\tau_nds-\Phi_0-\\-\mu\{\frac{1}{\sqrt{\pi^n}}\int_0^t\int_{R^n}\exp(-(\tau_1^2+\tau_2^2+\ldots+\tau_n^2))\sum_{j=1}^n\Phi_{0l_j^2}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},\ldots,x_n+\\+2\tau_n\sqrt{\mu(t-s)};s)d\tau_1d\tau_2\ldots d\tau_nds\}=\frac{1}{\sqrt{\pi^n}}\int_0^t\int_{R^n}\exp(-(\tau_1^2+\tau_2^2+\ldots+\tau_n^2))\sum_{j=1}^n\sqrt{\mu}\frac{\tau_j}{\sqrt{t-s}}\Phi_{0l_j}(x_1+\\+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},\ldots,x_n+2\tau_n\sqrt{\mu(t-s)};s)d\tau_1d\tau_2\ldots d\tau_nds-\\-\frac{1}{2}\sqrt{\mu}\{\frac{1}{\sqrt{\pi^n}}\int_0^t\frac{1}{\sqrt{t-s}}[\int_{R^n}\exp(-(\tau_1^2+\tau_2^2+\ldots+\tau_n^2))\Phi_{0l_1^2}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},\ldots,\\x_n+2\tau_n\sqrt{\mu(t-s)};s)d(x_1+2\tau_1\sqrt{\mu(t-s)})d\tau_2\ldots d\tau_n+\int_{R^n}\exp(-(\tau_1^2+\tau_2^2+\ldots+\tau_n^2))\Phi_{0l_2^2}(x_1+\\+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},\ldots,x_n+2\tau_n\sqrt{\mu(t-s)};s)d\tau_1d(x_2+2\tau_2\sqrt{\mu(t-s)})\ldots d\tau_n+\\+\int_{R^n}\exp(-(\tau_1^2+\tau_2^2+\ldots+\tau_n^2))\Phi_{0l_3^2}(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},\ldots,x_n+2\tau_n\sqrt{\mu(t-s)};s)\times\end{cases}$$

$$\begin{cases} \times d\tau_1 d\tau_2 \dots d(x_n + 2\tau_n\sqrt{\mu(t-s)})]ds\} = \frac{1}{\sqrt{\pi^n}}\int_0^t\int_{R^n} \exp(-(\tau_1^2+\tau_2^2+\dots+\tau_n^2))\sum_{j=1}^n \sqrt{\mu}\frac{\tau_j}{\sqrt{t-s}}\Phi_{0l_j}(x_1 + \\ +2\tau_1\sqrt{\mu(t-s)}, x_2 + 2\tau_2\sqrt{\mu(t-s)}, \dots, x_n + 2\tau_n\sqrt{\mu(t-s)}; s)d\tau_1 d\tau_2 \dots d\tau_n ds - \\ -\sqrt{\mu}\{\frac{1}{\sqrt{\pi^n}}\int_0^t \frac{1}{\sqrt{t-s}}\int_{R^n} \exp(-(\tau_1^2+\tau_2^2+\dots+\tau_n^2))\sum_{j=1}^n \tau_j \Phi_{0l_j}(x_1 + 2\tau_1\sqrt{\mu(t-s)}, x_2 + \\ +2\tau_2\sqrt{\mu(t-s)}, \dots, x_n + 2\tau_n\sqrt{\mu(t-s)}; s)d\tau_1 d\tau_2 \dots d\tau_n ds = 0. \end{cases}$$

That it was required to prove.

From the received results follows that functions υ_i are defined on the basis of (7.14), i.e.

$$\upsilon_i = \lambda_i[\vartheta_0(x_1, x_2, \dots, x_n) + H(x_1, x_2, \dots, x_n, t)], \forall(x_1, x_2, \dots, x_n, t) \in T_*, (i = \overline{1,n}). \qquad (7.19)$$

Further, considering partial derivatives of 1st order systems (7.19) and summing up with acceptance in attention $(1.2)_n$, (7.14) we have, that the system (7.19) satisfies to a condition $(1.2)_n$.

II. So as $\upsilon_{i0} \equiv \lambda_i \vartheta_0,\ (\vartheta_0 \in C^3(R^n))$, that the decision of problem $(1.1)_n$-$(1.3)_n$ belongs in $\tilde{C}_n^{3,1}(T_*)$:

$$\begin{cases} \|v\|_{\tilde{C}_n^{3,1}(T_*)} = \sum_{i=1}^n \{\sum_{0\le|k|\le 3} \|D^k \upsilon_i\|_{C(T_*)} + \|\upsilon_{it}\|_{C(T_*)}\}, \\ v = (\upsilon_1, \upsilon_2, \dots, \upsilon_n);\ \ \tilde{C}_n^{3,1}(T_*) \equiv \tilde{C}_n^{3,3,\dots,3,1}(T_*) \ne C_n^{3,3,\dots,3,1}(T_*), \\ \upsilon_i \equiv \lambda_i[\vartheta_0 + Z];\ \ \vartheta_0 \in C^3(R^n);\ \ \Delta\vartheta_0 = 0,\ (i = \overline{1,n}), \\ \|Z\|_{\tilde{C}^{3,1}(T_*)} = \sum_{0\le|k|\le 3} \|D^k Z\|_{C(T_*)} + \|Z_t\|_{C(T_*)}. \end{cases}$$

Really, if

$$\begin{cases} \upsilon_{i0} \in C^3(R^n); \Phi_0: \\ \sup_{R^n} |D^k \upsilon_{i0}| \le \gamma_1;\qquad \sup_{T_*} |D^k \Phi_0(x_1, x_2, \dots, x_n, t)| \le \beta_1,\ \ (i = \overline{1,n};\ k = \overline{0,3}), \\ \sup_{T_*} \frac{1}{\sqrt{\pi^n}}\int_0^t\int_{R^n} \exp(-(\tau_1^2+\tau_2^2+\dots+\tau_n^2)) |D^k \Phi_0(l_1, l_2, \dots, l_n; \tau)| d\tau_1 d\tau_2 \dots d\tau_n d\tau \le \beta_1 T_0 = \beta_2, \\ \sup_{T_*} \frac{1}{\sqrt{\pi^n}}\int_0^t\int_{R^n} \exp(-(\tau_1^2+\tau_2^2+\dots+\tau_n^2)) \frac{1}{\sqrt{t-\tau}}\sum_{j=1}^n |\tau_j| \times |\Phi_{0l_j^k}^{(k)}(l_1, l_2, \dots, l_n; \tau)| d\tau_1 d\tau_2 \dots d\tau_n d\tau \le \\ \le n\beta_1\sqrt{2T_0} = \beta_3,\ \ (l_j = x_j + 2\tau_j\sqrt{\mu(t-\tau)};\ j = \overline{1,n}), \\ \beta = \max_{1\le i\le 3}(\beta_i; \gamma_1),\ \ \beta_0 = \beta(1+\sqrt{\mu}), \end{cases} \qquad (7.20)$$

that on a basis (7.17) we will receive

$$\|Z\|_{\tilde{C}^{3,1}(T_*)} \le N_1 + \beta_0.$$

In the conclusion estimating (7.19) it is had

$$
\begin{cases}
v = (\upsilon_1, \upsilon_2, \dots, \upsilon_n); \quad \upsilon_i = \lambda_i[\vartheta_0 + Z], (i = \overline{1,n}): \\
\|v\|_{\tilde{C}_n^{3,1}(T_*)} = \sum_{i=1}^{n} [\|\lambda_i Z\|_{C^{3,\beta}(T_*)} + \|\lambda_i \vartheta_0\|_{C^3(T_*)} + \|\lambda_i Z_t\|_{C(T_*)}] \le d_0[N_1 + N_2 + \beta_0] = d_0[N_0 + \beta_0], \\
\|Z\|_{\tilde{C}^{3,1}(T_*)} = \|Z\|_{C^{3,\beta}(T_*)} + \|Z_t\|_{C(T_*)} \le N_1 + \beta_0, \ (d_0 = \sum_{i=1}^{n} \lambda_i; \ N_0 = N_1 + N_2).
\end{cases}
$$

Lemma 3. In the conditions of $(1.2)_n$, (7.13), (7.14) and (7.20) the equation (7.17) has a single solution in $\tilde{C}^{3,1}(T_*)$.

Theorem 8*. At performance of conditions of the lemma 3 the problem $(1.1)_n$, $(1.2)_n$, (7.13) has a smooth single solution in $\tilde{C}_n^{3,1}(T_*)$ of the defined by a rule (7.19).

The essential factor of researches of this paragraph are results of the theorem 8*. In this case the decision of system $(1.1)_n$ is considered as the strict solution of a problem $(1.1)_n$-$(1.3)_n$ in $\tilde{C}_n^{3,1}(T_*)$.

It is obvious that small changes $\upsilon_{i0}, (i = \overline{1,n})$ or $f_i, (i = \overline{1,n})$ influence the decision (7.17) a little, i.e. continuous depends on this data. Therefore, a question on a statement correctness problems $(1.1)_n$-$(1.3)_n$ are considered at once with results of the theorem 8*.

7.3. Fluid with Average and with a Great Number of Viscosity, when $f_i \equiv 0, (i = \overline{1,3})$

In this paragraph we generalise results of 5.3, so as this cases has applied value in the theory of a liquid of average and small currents [12], when:

$$
(x,t) \in T_* = R^n \times [0,T_0]; \ f_i \equiv 0, (i = \overline{1,n}), \ 0 < n_0 \le \mu = \mu_0 < \infty, \ (n_0, \mu_0 = \text{const})). \tag{7.21}
$$

Under a condition (7.21) the problem $(1.1)_n$ – $(1.3)_n$ is led to a kind

$$
\frac{\partial \upsilon_i}{\partial t} + \sum_{j=1}^{n} \upsilon_j \frac{\partial \upsilon_i}{\partial x_j} = \mu \Delta \upsilon_i - \frac{1}{\rho} P_{x_i}, (i = \overline{1,n}), \tag{7.22}
$$

$$
\operatorname{div} v = 0, \forall (x,t) \in T_*, \tag{7.23}
$$

$$
\begin{cases}
\upsilon_i |_{t=0} = \lambda_i \vartheta_0(x_1, \dots, x_n), \forall (x_1, \dots, x_n) \in R^n, \\
\upsilon_{i0} \equiv \lambda_i \vartheta_0, \ (\vartheta_0 \in R^n; \ i = \overline{1,n}),
\end{cases} \tag{7.24}
$$

where $0 < \lambda_i -$ the known constants. Hence, here we will consider a methods of the equations integration of Navier-Stokes (7.22) with a conditions (7.23), (7.24).

With that end in view we will assume that there are functions $0 \le f_{i\delta}, (i = \overline{1,n})$, which satisfy conditions

$$\begin{cases} C^{3,0}(T_*) \ni f_{i\delta}: \quad \sup_{T_*}\left|D^k f_{i\delta}\right| \le \alpha_i(\delta) \le \alpha(\delta) < 1, \ (0 < \delta < 1;\ i = \overline{1,n}), \\ \operatorname{div} f_\delta = 0, \quad \operatorname{rot} f_\delta = 0: \\ \Delta f_{i\delta} = 0; \ (f_\delta = (f_{1\delta}, f_{2\delta}, \dots, f_{n\delta})), \\ I_{i\delta}(x_1, x_2, \dots, x_n, t) \equiv \int_0^t f_{i\delta}(x_1, x_2, \dots, x_n, s')ds', \ (\Delta I_{i\delta} = 0;\ I_{i\delta t} \equiv f_{i\delta};\ i = \overline{1,n}). \end{cases} \tag{7.25}$$

Hence is offered the method:

$$\begin{cases} \upsilon_i = \lambda_i[\vartheta_0 + Z(x_1, \dots, x_n, t)] + I_{i\delta}(x_1, \dots, x_n, t), (i = \overline{1,n}), \\ Z\big|_{t=0} = 0, \forall (x_1, \dots, x_n) \in R^n, \\ \operatorname{div} v = 0: \ \sum_{i=1}^n \lambda_i \vartheta_{0x_i} = 0; \ \sum_{i=1}^n \lambda_i Z_{x_i} = 0; \ \sum_{i=1}^n I_{i\delta x_i} = 0, \\ \vartheta_0 \in R^n, \Delta\vartheta_0 = 0, \ (i = \overline{1,n}). \end{cases} \tag{7.26}$$

Thus takes place conditions

$$\begin{cases} \sum_{i=1}^n \frac{\partial}{\partial x_i} I_{i\delta t} = 0, \\ \sum_{j=1}^n \upsilon_j \upsilon_{ix_j} \equiv \sum_{j=1}^n \lambda_j(\vartheta_0 + Z) I_{i\delta x_j} + \sum_{j=1}^n I_{j\delta}\lambda_i(\vartheta_{0x_j} + Z_{x_j}) + \sum_{j=1}^n I_{j\delta} I_{i\delta x_j}, \\ \sum_{j=1}^n \lambda_j(\vartheta_0 + Z)\lambda_i(\vartheta_{0x_j} + Z_{x_j}) = \lambda_i(\vartheta_0 + Z)\sum_{j=1}^n \lambda_j(\vartheta_{0x_j} + Z_{x_j}) = 0, \\ \upsilon_{it} \equiv \lambda_i Z_t + I_{i\delta t}; \ \mu\Delta\upsilon_i \equiv \mu[\lambda_i(\Delta\vartheta_0 + \Delta Z) + \Delta I_{i\delta}] = \mu\lambda_i \Delta Z, (i = \overline{1,n}). \end{cases} \tag{7.27}$$

Then for incompressible currents with a friction the equations of Navier-Stokes (7.22) become simpler as take place (7.23), (7.24). Therefore the problem (7.22)-(7.24), is led to a kind

$$\lambda_i Z_t + \sum_{j=1}^n \lambda_j(\vartheta_0 + Z) I_{i\delta x_j} + \sum_{j=1}^n I_{j\delta}\lambda_i(\vartheta_{0x_j} + Z_{x_j}) + \sum_{j=1}^n I_{j\delta} I_{i\delta x_j} = -f_{i\delta} - \frac{1}{\rho}P_{x_i} + \mu\lambda_i\Delta Z, (i = \overline{1,n}). \tag{7.28}$$

From system (7.28), considering conditions (7.22)-(7.24) and having entered [8]: APS, we will receive the equation:

$$\begin{cases} \operatorname{div} f_\delta = 0; \quad \sum_{i=1}^n \frac{\partial}{\partial x_i}(7.28): \\ \Delta \frac{1}{\rho} P = -\{F_{0\delta} + B_\delta[Z_{x_1}, Z_{x_2}, \dots, Z_{x_n}]\}, \\ (B_\delta[Z_{x_1}, Z_{x_2}, \dots, Z_{x_n}])(x_1, \dots, x_n, t) \equiv \sum_{i=1}^n (\sum_{j=1}^n \lambda_j I_{i\delta x_j}) Z_{x_i} + \sum_{i=1}^n (\sum_{j=1}^n I_{j\delta x_i} Z_{x_j})\lambda_i, \\ F_{0\delta}(x_1, \dots, x_n, t) \equiv \sum_{i=1}^n \lambda_i (\sum_{j=1}^n I_{j\delta x_i}\vartheta_{0x_j}) + \sum_{i=1}^n \vartheta_{0x_i}(\sum_{j=1}^n \lambda_j I_{i\delta x_j}) + \sum_{i=1}^n (\frac{\partial}{\partial x_i}(\sum_{j=1}^n I_{j\delta} I_{i\delta x_j})), \end{cases}$$

$$\begin{cases} \frac{1}{\rho} P = \frac{1}{4\pi} \int\limits_{R^n} \frac{1}{r} \{ F_{0\delta}(s_1, s_2, \dots, s_n, t) + (B_\delta [Z_{s_1}, Z_{s_2}, \dots, Z_{s_n}])(s_1, s_2, \dots, s_n, t) \} ds_1 ds_2 \dots ds_n, \\ \frac{1}{\rho} P_{x_i} = \frac{1}{4\pi} \int\limits_{R^n} \frac{\tau_i}{\sqrt{(\tau_1^2 + \tau_2^2 + \dots + \tau_n^2)^3}} \{ F_{0\delta}(x_1 + \tau_1, x_2 + \tau_2, \dots, x_n + \tau_n; t) + (B_\delta [Z_{h_1}, Z_{h_2}, \dots, Z_{h_n}])(x_1 + \\ + \tau_1, x_2 + \tau_2, \dots, x_n + \tau_n; t) \} d\tau_1 d\tau_2 \dots d\tau_n, \ \ (s_i - x_i = \tau_i; \ h_i = x_i + \tau_i; \ \ i = \overline{1,n}), \end{cases} \tag{7.29}$$

so as takes place

$$\begin{cases} \sum_{i=1}^{n} \frac{\partial}{\partial x_i} \{ \sum_{j=1}^{n} \lambda_j (\vartheta_0 + Z) I_{i\delta x_j} + \sum_{j=1}^{n} I_{j\delta} \lambda_i (\vartheta_{0x_j} + Z_{x_j}) \} = \sum_{i=1}^{n} \lambda_i (\sum_{j=1}^{n} I_{j\delta x_i} \vartheta_{0x_j}) + \sum_{i=1}^{n} \vartheta_{0x_i} (\sum_{j=1}^{n} \lambda_j I_{i\delta x_j}) + \\ + \sum_{i=1}^{n} (\sum_{j=1}^{n} \lambda_j I_{i\delta x_j}) Z_{x_i} + \sum_{i=1}^{n} (\sum_{j=1}^{n} I_{j\delta x_i} Z_{x_j}) \lambda_i, \\ (\sum_{j=1}^{n} I_{j\delta x_j})_{x_i} = 0; \ (\sum_{j=1}^{n} \lambda_j \vartheta_{0x_j})_{x_i} = 0, \ (\sum_{j=1}^{n} \lambda_j Z_{x_j})_{x_i} = 0, \ (i = \overline{1,n}), \\ \frac{\partial}{\partial t} [\sum_{i=1}^{n} \lambda_i Z_{x_i}] = 0, \\ \mu \sum_{i=1}^{n} \frac{\partial}{\partial x_i} (\lambda_i \Delta Z) = 0; \quad \sum_{i=1}^{n} \frac{\partial}{\partial x_i} (-\frac{1}{\rho} P_{x_i}) \equiv -\frac{1}{\rho} \Delta P. \end{cases}$$

Then on a basis (7.29) the system (7.28) it is equivalent, will be transformed to a kind:

$$\begin{cases} Z_t = \Phi_{0\delta} + (Q[Z, Z_{x_1}, Z_{x_2}, \dots, Z_{x_n}])(x_1, \dots, x_n, t) + \mu \Delta Z, (i = \overline{1,n}), \\ Z \big|_{t=0} = 0, \forall (x_1, \dots, x_n) \in R^n, \\ \Phi_{0\delta}(x_1, \dots, x_n, t) \equiv -\sum_{j=1}^{n} I_{j\delta} \vartheta_{0x_j} + d_0^{-1} [\sum_{i=1}^{n} (-f_{i\delta}) - \sum_{i=1}^{n} \vartheta_0 (\sum_{j=1}^{n} \lambda_j I_{i\delta x_j}) - \sum_{i=1}^{n} (\sum_{j=1}^{n} I_{j\delta} I_{i\delta x_j}) - \\ - \frac{1}{4\pi} \int\limits_{R^n} \{ \sum_{i=1}^{n} \frac{\tau_i}{\sqrt{(\tau_1^2 + \tau_2^2 + \dots + \tau_n^2)^3}} (F_{0\delta}(x_1 + \tau_1, x_2 + \tau_2, \dots, x_n + \tau_n; t)) \} d\tau_1 d\tau_2 \dots d\tau_n], \\ (Q[Z, Z_{x_1}, Z_{x_2}, \dots, Z_{x_n}])(x_1, \dots, x_n, t) \equiv -\{ d_0^{-1} Z(x_1, \dots, x_n, t) [\sum_{i=1}^{n} (\sum_{j=1}^{n} \lambda_j I_{i\delta x_j}(x_1, \dots, x_n, t))] + \\ + \sum_{j=1}^{n} Z_{x_j}(x_1, \dots, x_n, t) I_{j\delta}(x_1, \dots, x_n, t) + d_0^{-1} [\frac{1}{4\pi} \int\limits_{R^n} (\sum_{i=1}^{n} \frac{\overline{\tau}_i}{\sqrt{(\overline{\tau}_1^2 + \overline{\tau}_2^2 + \dots + \overline{\tau}_n^2)^3}} \{ (B_\delta [Z_{\overline{h}_1}, \dots, Z_{\overline{h}_n}])(x_1 + \\ + \overline{\tau}_1, x_2 + \overline{\tau}_2, \dots, x_n + \overline{\tau}_n; t) \}) d\overline{\tau}_1 d\overline{\tau}_2 \dots d\overline{\tau}_n] \}, \\ \overline{h}_1 = x_i + \overline{\tau}_i, \ \ (i = \overline{1,n}); \ \ d_0 = \sum_{i=1}^{n} \lambda_i > 0. \end{cases} \tag{7.30}$$

The problem (7.30) is led to system of the integrated equations in a kind

$$\begin{cases} Z_{x_i} = W_i(x_1, \dots, x_n, t), \forall (x_1, \dots, x_n, t) \in T, (i = \overline{1,n}), \\ Z = M_{1\delta} + \frac{1}{2^n \sqrt{\pi^n}} \int\limits_0^t \int\limits_{R^n} \exp(-\frac{r^2}{4\mu(t-s)}) \frac{1}{(\sqrt{\mu(t-s)})^n} (Q[Z, W_1, W_2, \dots, W_n])(s_1, s_2, \dots, s_n, s) \times \end{cases}$$

$$\begin{cases} \times ds_1 ds_2 \ldots ds_n ds = M_{1\delta} + \dfrac{1}{\sqrt{\pi}^n}\int\limits_0^t \int\limits_{R^n} \exp(-(\tau_1^2 + \tau_2^2 + \ldots + \tau_n^2))(Q[Z,W_1,W_2,\ldots,W_n])(x_1 + 2\tau_1 \times \\ \times\sqrt{\mu(t-s)}, x_2 + 2\tau_2\sqrt{\mu(t-s)},\ldots,x_n + 2\tau_n\sqrt{\mu(t-s)}; s)d\tau_1 d\tau_2\ldots d\tau_n ds \equiv \\ \equiv (\Gamma_0[Z,W_1,W_2,\ldots,W_n])(x_1,\ldots,x_n,t), \\ W_i = M_{1\delta x_i} + \dfrac{1}{2^n\sqrt{\pi}^n}\int\limits_0^t\int\limits_{R^n}(\exp(-\dfrac{r^2}{4\mu(t-s)}))\dfrac{-(x_i - s_i)}{2\mu(t-s)}\dfrac{1}{(\sqrt{\mu(t-s)})^n}(Q[Z,W_1,W_2,\ldots,W_n])\times \\ \times(s_1,s_2,\ldots,s_n,s)ds_1 ds_2\ldots ds_n ds = M_{1\delta x_i} + \dfrac{1}{\sqrt{\pi}^n}\int\limits_0^t\int\limits_{R^n}\exp(-(\tau_1^2+\tau_2^2+\ldots+\tau_n^2))\dfrac{\tau_i}{\sqrt{\mu(t-s)}}\times \\ \times(Q[Z,W_1,W_2,\ldots,W_n])(x_1 + 2\tau_1\sqrt{\mu(t-s)},x_2 + 2\tau_2\sqrt{\mu(t-s)},\ldots,x_n + 2\tau_n\sqrt{\mu(t-s)};s)\times \\ \times d\tau_1 d\tau_2\ldots d\tau_n ds \equiv (\Gamma_i[Z,W_1,W_2,\ldots,W_n])(x_1,\ldots,x_n,t),\ (s_i - x_i = 2\tau_i\sqrt{\mu(t-s)}; i = \overline{1,n}), \\ M_{1\delta}(x_1,\ldots,x_n,t) \equiv \dfrac{1}{\sqrt{\pi}^n}\int\limits_0^t\int\limits_{R^n}\exp(-(\tau_1^2+\tau_2^2+\ldots+\tau_n^2))\Phi_{0\delta}(x_1 + 2\tau_1\sqrt{\mu(t-s)},x_2 + 2\tau_2\times \\ \times\sqrt{\mu(t-s)},\ldots,x_n + 2\tau_n\sqrt{\mu(t-s)};s)d\tau_1 d\tau_2\ldots d\tau_n ds. \end{cases} \tag{7.31}$$

To solve a problem (7.30) concerning this problem we will receive system (7.31) of four integral equations.

Let concerning known functions $M_{1\delta}, \Pi_\delta, I_{i\delta}$ takes place:

$$\begin{cases} \forall(x_1,\ldots,x_n,t)\in T_*; M_{1\delta},\Pi_\delta,I_{i\delta}:\ \sup\limits_{T_*}\left|D^k M_{1\delta}(x_1,x_2,x_3,t)\right| \le \beta_1\alpha(\delta),\ (k = \overline{0,3};\ \ t - s = \tau), \\ \sup\limits_{T_*\times T_*}\Pi_\delta(x_1,\ldots,x_n,\tau_1,\ldots,\tau_n;t,\tau) \equiv \sup\limits_{T_*\times T_*}\{d_0^{-1}\sum\limits_{i=1}^n(\sum\limits_{j=1}^n\lambda_j\left|I_{i\delta s_j}(x_1 + 2\tau_1\sqrt{\mu\tau},x_2 + 2\tau_2\sqrt{\mu\tau},\ldots,x_n + \right. \\ \left. +2\tau_n\sqrt{\mu\tau};t-\tau)\right|) + \sum\limits_{j=1}^n\left|I_{j\delta}(x_1 + 2\tau_1\sqrt{\mu\tau},x_2 + 2\tau_2\sqrt{\mu\tau},\ldots,x_n + 2\tau_n\sqrt{\mu\tau};t-\tau)\right| + \\ +d_0^{-1}[\dfrac{1}{4\pi}\int\limits_{R^n}(\sum\limits_{i=1}^n\dfrac{\left|\overline{\tau}_i\right|}{\sqrt{(\overline{\tau}_1^2 + \overline{\tau}_2^2 + \ldots + \overline{\tau}_n^2)^3}}\{\sum\limits_{i=1}^n(\sum\limits_{j=1}^n\lambda_j\left|I_{i\delta l_j}(x_1 + 2\tau_1\sqrt{\mu\tau} + \overline{\tau}_1,x_2 + 2\tau_2\sqrt{\mu\tau} + \right. \\ \left. +\overline{\tau}_2,\ldots,x_n + 2\tau_n\sqrt{\mu\tau} + \overline{\tau}_n;t-\tau)\right|) + \sum\limits_{i=1}^n(\sum\limits_{j=1}^n\left|I_{j\delta l_i}(x_1 + 2\tau_1\sqrt{\mu\tau} + \overline{\tau}_1,x_2 + 2\tau_2\sqrt{\mu\tau} + \overline{\tau}_2,\ldots,x_n + \right. \\ \left. +2\tau_n\sqrt{\mu\tau} + \overline{\tau}_n;t-\tau)\right|)\lambda_i\})d\overline{\tau}_1 d\overline{\tau}_2\ldots d\overline{\tau}_n]\} \le \beta_2\alpha(\delta), \\ k_0 = \dfrac{1}{\sqrt{\pi}^n}\sup\limits_{T_*}\int\limits_0^t\int\limits_{R^n}\exp(-(\tau_1^2+\tau_2^2+\ldots+\tau_n^2))\Pi_\delta(x_1,\ldots,x_n,\tau_1,\ldots,\tau_n;t,\tau)d\tau_1\ldots d\tau_n d\tau \le \beta_2\alpha(\delta)T_0, \\ k_i = \dfrac{1}{\sqrt{\pi}^n}\sup\limits_{T_*}\int\limits_0^t\int\limits_{R^n}\exp(-(\tau_1^2+\tau_2^2+\ldots+\tau_n^2))\Pi_\delta(x_1,\ldots,x_n,\tau_1,\ldots,\tau_n;t,\tau)\dfrac{\left|\tau_i\right|}{\sqrt{\mu\tau}}d\tau_1\ldots d\tau_n d\tau \le \\ \le\sqrt{2T_0}\beta_2\alpha(\delta)\dfrac{1}{\sqrt{\mu}},(i = \overline{1,n}), \\ \beta = \max(\beta_2 T_0;\ n\sqrt{2T_0}\beta_2), \end{cases} \tag{7.32}$$

and if operators: $\Gamma_i, (i = \overline{0,n})$ compressing with a compression factor k_i,

$$\begin{cases} \Gamma_i,(i=\overline{0,n}):\ k_i \le \dfrac{h}{n+1},(h<1),\ (i=\overline{1,n}), \\ \sum\limits_{i=0}^{n} k_i \le \alpha(\delta)(\beta_2 T_0 \dfrac{1}{\sqrt{\mu}} + n\sqrt{2T_0}\beta_2) \le \alpha(\delta)(\dfrac{1}{\sqrt{n_0}}+1)\beta = h < 1, \\ \alpha(\delta) < \delta[(\dfrac{1}{\sqrt{n_0}}+1)\beta]^{-1},\ (0<\delta<1;\ \ 0<n_0\le\mu=\mu_0<\infty), \\ S_{r_1}(0)=\{Z,W_i : |Z|;\ |W_i| \le r_1, \forall(x_1,\ldots,x_n,t)\in T_*\},(i=\overline{1,n}), \end{cases} \tag{7.33}$$

and:

$$\begin{cases} \|\Gamma_i[0,0,0,\ldots,0]\|_C \le r_1(1-h): \\ \|\Gamma_i[Z,W_1,W_2,\ldots,W_n]\|_C \le \|\Gamma_i[Z,W_1,W_2,\ldots,W_n]-\Gamma_i[0,0,0,\ldots,0]\|_C + \|\Gamma_i[0,0,0,\ldots,0]\|_C \le \\ \le k_i(n+1)r_1 + r_1(1-h) \le hr_1 + r_1(1-h) = r_1, \\ \Gamma_i : S_{r_1}(0) \to S_{r_1}(0),\ (i=\overline{0,n}). \end{cases} \tag{7.34}$$

Hence on the basis of contraction mapping principle system (7.31) is solvable and for which makes Picard’s method

$$\begin{cases} \forall(x_1,\ldots,x_n,t)\in T_*:\quad Z_{m+1} = \Gamma_0[Z_m,W_{1,m},W_{2,m},\ldots,W_{n,m}], \\ W_{i,m+1} = \Gamma_i[Z_m,W_{1,m},W_{2,m},\ldots,W_{n,m}],(m=0,1,\ldots;Z_0=0;\ \ W_{i,0}=0;\ i=\overline{1,n}). \end{cases} \tag{7.35}$$

Received the sequence of functions $\{Z_m\}_0^\infty,\{W_{i,m}\}_0^\infty$ is converging and fundamental in $S_{r_1}(0)$, i.e.:

$$\begin{cases} E_{m+1} = \|Z_{m+1}-Z_m\|_C + \sum\limits_{i=1}^{n}\|W_{i,m+1}-W_{i,m}\|_C;\ E_m = \|Z_m - Z_{m-1}\|_C + \sum\limits_{i=1}^{n}\|W_{i,m}-W_{i,m-1}\|_C : \\ \|Z_{m+1}-Z_m\|_C \le k_0 E_m;\ \ \|W_{i,m+1}-W_{i,m}\|_C \le k_i E_m,(i=\overline{1,n}), \\ E_{m+1} \le hE_m \le \ldots \le h^m E_1 \xrightarrow[m\to\infty]{h<1} 0, \\ \|Z_{m+k}-Z_m\|_C \le \sum\limits_{j=0}^{k-1} k_0 E_{m+j};\quad \|W_{i,m+k}-W_{i,m}\|_C \le \sum\limits_{j=0}^{k-1} k_i E_{m+j},(i=\overline{1,n}), \\ E_{m+k} \le h\sum\limits_{j=0}^{k-1} E_{m+j} \le \ldots \le h\sum\limits_{j=0}^{k-1} h^{m+j-1}E_1 \le E_1 h^m \sum\limits_{j=0}^{k-1} h^j \le E_1 h^m \dfrac{1}{1-h} \xrightarrow[m\to\infty]{h<1} 0, \\ X_0 = \|Z\|_C + \sum\limits_{i=1}^{n}\|W_i\|_C;\ X_{m+1} \equiv \|Z_{m+1}-Z\|_C + \sum\limits_{i=1}^{n}\|W_{i,m+1}-W_i\|_C : \\ X_{m+1} \le h^{m+1}X_0 \xrightarrow[m\to\infty]{h<1} 0, \\ Z_{m+1} \xrightarrow[m\to\infty]{h<1} Z \equiv H;\ \ W_{i,m+1} \xrightarrow[m\to\infty]{h<1} W_i, \forall(x_1,\ldots,x_n,t)\in T_*,(i=\overline{1,n}). \end{cases} \tag{7.36}$$

Therefore on the basis of (7.26) and

$$\upsilon_{i,m+1} = \lambda_i[\vartheta_0(x_1,\ldots,x_n) + Z_{m+1}(x_1,\ldots,x_n,t)] + I_{i\delta}(x_1,\ldots,x_n,t),(m=0,1,2,\ldots;i=\overline{1,n}), \tag{7.37}$$

we will receive

$$\|\upsilon_{i,m+1}-\upsilon_i\|_C \le \lambda_i\|Z_{m+1}-Z\|_C \le \lambda_i h X_m \le \lambda_i h^{m+1}X_0 \xrightarrow[m\to\infty]{h<1} 0,(i=\overline{1,n}).$$

And it means that sequence $\{\upsilon_{i,m}\}_0^\infty$ converging to a limit $\upsilon_i,(i=\overline{1,n}):$

$$\upsilon_{i,m+1}\xrightarrow[m\to\infty]{h<1}\upsilon_i\in\tilde{C}^{3,1}(T_*),(i=\overline{1,n}). \qquad (7.38)$$

Theorem 9. If conditions (7.23), (7.24), (7.25), (7.33), (7.34) and (7.38) are executed, that problem Navier-Stokes is correctly put in $\tilde{C}_n^{3,1}(T_*)$.

□ **Proof.** It is received «$n+1$» - square system of the integral equations. It is thus proved that for this system all conditions of contraction mapping principle are realised. Then the system (7.31) is unequivocally compatible, i.e. on a basis (7.36) there is the smooth and single function $Z(x_1,\ldots,x_n,t)\in\tilde{C}^{3,1}(T_*)$. Hence considering (7.26), under conditions (7.23), (7.24), (7.25), (7.33), (7.34) and (7.38) we will receive $\upsilon_i\in\tilde{C}^{3,1}(T_*),(i=\overline{1,n})$.

It is obvious that small changes $\upsilon_{i0}\equiv\lambda_1\vartheta_0,(i=\overline{1,n})$ or $f_{i\delta},(i=\overline{1,n})$ influence the decision (7.26) a little, i.e. continuous depends on this data. Therefore, a question on a statement correctness problems (7.22)-(7.24) are considered at once with results of the theorem 9 in $\tilde{C}_n^{3,1}(T_*)$. ■

VIII. Conclusion

Analytical solution of Navier-Stokes equation for incompressible fluid is the 6th millennium problem. The difficulty of Navier-Stokes equation solution is caused by nonlinear nature and increased by the necessity to find speed and pressure, depending on any values of viscosity as a parameter, factor in spatial 3D problem.

The essence of the given work consists that the developed methods of an initial problem lead to Navier-Stokes equations linearization into integral form without attraction of any conditions for solving the task (1.1)-(1.3). That's why the received analytical solution is regular to viscosity factor, and mathematic meanings of variables respond to physical significance according to requirements of Clay Mathematics Institute. So, The developed out in this article methods of solution Navier Stokes equations bring the task to equivalent linear equations with heat conductivity type under Cauchy condition in class of functions with smooth enough initial conditions under $t=0$. Exactly in such a class there is only smooth or conditionally-smooth solution of Navier-Stokes task in $\tilde{C}_{n=3}^{3,1}(T)$ or $G_{n=3}^1(D_0),(\text{or } W_\lambda^2(D_0))$.

We should remind that Clay Institute suggested solving one from three variants of Navier Stokes equation. But in this work summarising the result received in article [8] for any viscosity meanings in unlimited area in both spaces, as exactly this variant according to Clay Institute is more preferable in comparison with other variants received in [8].

The important appendix to solution appeared the research of fluid with very small viscosity (Reynolds number is great: **Re** ≥ 2300). It turned out that in this case analytical methods give the solution of Navier-Stokes problem and that allows to pass to turbulence understanding which was studied by Reinolds in 1876.

In addition in work are investigated and fluid with medium viscosity [12], in which all inertia members are preserved. That is the significance of the results as the solution of Navier-Stokes equations for incompressible fluid with medium viscosity served as litmus paper of correctness a method and proved value of method.

The solution of 6th millennium problem allowed to consider it that results of this work in $\tilde{C}^{3,1}_{n=3}(T)$ as principle of continuation of strong solutions connected with Beale-Kato-Majda [2, 5] criterion requirements but already for 3D Navier-Stokes equation for viscous incompressible fluid [1]. When analytical solution of Navier-Stokes equation is found so Beale-Kato-Majda's criterion for Euler's equation is automatically proved because viscosity meaning $\mu = 0$ in them responds to Euler's equation for ideal liquid.

From the received results follows the Navier-Stokes equations (1.1) in the conditions of (1.2), (1.3) have the smooth and single solution in $\tilde{C}^{3,1}_{n=3}(T),(\text{or } \tilde{C}^{3,1}_{n}(T_*))$, that reflect the requirements of 6th millennium problem [1].

REFERENCES

[1] *Navier-Stokes Existence and Smoothness Problem*. The Millennium Problems, stated in 2000 by Clay Mathematics Institute.

[2] Beale, J.T., Kato, T., Majda, A. (1984), *Remarks on the breakdown of smooth solutions for the 3-D Euler equations*, Comm. Math. Phys. 94 (1), pp. 61-66.

[3] Birkhoff, G. (1983), *Numerical fluid dynamics*. SIAM Rev., Vol. 25, No 1, pp. 1-34.

[4] Cantwell, B.J. (1981), *Organized motion in turbulent flow*. Ann. Rev. Fluid Mech. Vol. 13, pp. 457-515.

[5] Grujic, Z., Guberovic, R. (2010), *A regularity criterion for the 3D NSE in a local version of the space of functions of bounded mean oscillations*, Ann. Inst. Henri Poincare, Anal. Non Lineaire **27**, pp. 773-778.

[6] Ladyzhenskya, O.A. (1970), *Mathematical questions of dynamics of a viscous incompressible liquid* (in Russian). Nauka, Moscow, 288 p.

[7] Omurov, T.D. (2013), *Nonstationary Navier-Stokes Problem for Incompressible Fluid with Viscosity*. American J. Math.&Statistics, Vol. 3, No 6, pp. 349-356. **(**http://article.sapub.org/10.5923.j.ajms.20130306.08.html**)**

[8] Omurov, T.D. (2014), *The Methods of a Problem Decision Navier-Stokes for the Incompressible Fluid with Viscosity*. American J. of Fluid Dynamics, Vol. 4, No 1, pp. 16-48 **(**http://article.sapub.org/10.5923.j.ajfd.20140401.03.html **)**

[9] Omurov, T.D. (2013), *Navier-Stokes problem for Incompressible fluid with viscosity*. Varia Informatica, 2013, Ed. M.Milosz, PIPS Polish Lublin, pp. 137-158.

[10] Omurov, T.D. (2010), *Nonstationary Navier-Stokes Problem for Incompressible Fluid*. J.Balasagyn KNU, Bishkek, 21p. [Content of the work is registered in Kyrgyzpatent, and the copyright certificate is received]

[11] Prantdl, L. (1961), *Gesammelte Abhandlungen zur angewandten Mechanik, Hudro- und Aerodynamic*. Springer, Berlin.

[12] Schlichting's, H. (1974), *Boundary-Layer Theory*. Nauka, Moscow, 712 p.

[13] Sobolev, L.S. (1966), *Equations of Mathematical Physics*. Nauka, Moscow, 443 p.

[14] Friedman, A. (1958), *Boundary estimates for second order parabolic equations and their application*. J. of Math.and Mech., Vol. **7**, No 5, pp.771-791.

[15] Hörmander, L. (1985), *The Analysis of Linear Partial Differential Operators III: Pseudo - Differential Operators*. Springer-Verlag, Berlin Heidelberg, NY, Tokyo, 696 p.